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Primitive Central Idempotents of Nilpotent Group Algebras

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Abstract. We exhibit the primitive central idempotents of a semisimple group algebra of a finite nilpotent group over an arbitrary field (without using group characters), examining the abelian case separately. Our result extends and improves the main result in [**1**].

Introduction

Let G be a finite abelian group of order n, and K be a field such that $char(K)$ does not divide n. It is a well know result (see [**5**]), that the abelian group algebra KG is isomorphic to the direct sum $\bigoplus_i K(\zeta_i)$, where ζ_i are primitive roots of unity which orders divide n. Thus, clearly, the primitive idempotents of KG are the inverse images of each tuple of the form $(0, \ldots, 0, 1, 0, \ldots, 0)$ under the isomorphism above mentioned. We shall exhibit the primitive idempotents of KG and, in particular, we obtain the cyclic codes over finite fields.

Now, let G be an arbitrary finite group. A well known result is that the primitive central idempotents of the complex group algebra $\mathbb{C}G$ are all elements of the form $\frac{\chi(1)}{|G|} \sum_{g \in G} \chi(g^{-1})g$, where χ is an irreducible complex character of G and 1 is the identity of G (see [**6**]). With the known methods, computing the character table of a finite group is a task having complexity of exponential growth with respect to the order of the group. Therefore, alternative methods for computing the primitive central idempotents of $\mathbb{C}G$ are always of interest.

Consider now G a finite nilpotent group. The primitive central idempotents in a rational group algebra of a nilpotent group have been determined in [**1**], without making use of the character table of G. These were extended and simplified in [**4**], providing an algorithm using only elementary methods for calculating the primitive central idempotents of $\mathbb{Q}G$, when G is a finite nilpotent group, among other cases, but not of KG for an arbitrary field K. These improvements were implemented in a package [**2**] of programs for GAP System, version 4. An experimental comparison of the speed of the algorithm in [**2**] and the character method (computing primitive

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central idempotents from the character table of the group) was presented in [**3**] and showed that the first is usually faster. These improvements, however, do not carry on automatically to the case of KG.

We are going to use a method similar to the one presented in [**1**] and the abelian case in order to find out the primitive central idempotents in the group algebra of a finite nilpotent group over an arbitrary field K, provided that $char(K)$ does not divide $|G|$. Thus, we extend the result in [**1**] and improve it a little (since one of the conditions in our main theorem is slightly simpler than the one in [**1**]). Our description also allows the construction of the character table of a finite nilpotent group G using a lattice of subnormal subgroups of G, constructed in order to satisfy certain conditions.

1. The Abelian Case

We exhibit explicitly all the primitive idempotents in the semisimple group algebra of a finite abelian group.

Let K be a field, for each $(n_1, \ldots, n_s)$ an $s-$tuple of positive integers let $m = lcm(n_1, \ldots, n_s)$. Suppose that $char(K)$ does not divide m. Consider ζ_i a primitive root of unity of order n_i , for $i = 1, \ldots, s$. Given $\bar{l} = (l_1, \ldots, l_s)$ an s-tuple of integers, with $0 \leqslant l_i \leqslant n_i - 1$, define the polynomial:

$$P_{\bar{l}} = \prod_{i=1}^{s} \prod_{k_i=0, k_i \neq l_i}^{n_i-1} (X_i - \zeta_i^{k_i}) \in K(\zeta_m)[X_1, \ldots, X_s] \tag{1.1}$$

Notice that $P_{\bar{l}}(\zeta_1^{k_1}, \ldots, \zeta_s^{k_s}) \neq 0$ iff $\overline{k} = \bar{l}$.

Let $G \simeq C_1 \times \ldots \times C_s$ be a finite abelian group of order n, with C_i a cyclic group of order n_i generated by g_i, and let K be an algebraically closed field such that char(K) does not divide n. The group algebra KG is isomorphic to the direct sum $K \oplus \ldots \oplus K$ of n copies of K, this isomorphism maps $(1, \ldots, 1, g_i^{k_i}, 1, \ldots, 1)$ to

$$(1, \ldots, 1, \zeta_{n_i}^{k_i}, \ldots, \zeta_{n_i}^{k_i}, \zeta_{n_i}^{k_i 2}, \ldots, \zeta_{n_i}^{k_i 2}, \ldots, \zeta_{n_i}^{k_i(n_i-1)}, \ldots, \zeta_{n_i}^{k_i(n_i-1)}),$$

and in general:

$$(g_1^{k_1}, \ldots, g_s^{k_s}) \mapsto (\ldots, \zeta_1^{k_1 l_1} \ldots \zeta_s^{k_s l_s}, \ldots)_{0 \leqslant l_i \leqslant n_i - 1}, \tag{1.2}$$

where $\zeta_i \in K$ is a primitive root of unity of order n_i for each $i = 1, \ldots, s$.

Theorem 1.1. *Let $G \simeq C_1 \times \ldots \times C_s$ be a finite abelian group of order n, with C_i a cyclic group of order n_i generated by g_i, and let K be an algebraically closed field such that $char(K)$ does not divide n. Then the primitive idempotents of the abelian group algebra KG are the elements:*

$$e_{\bar{l}} := \frac{P_{\bar{l}}(g_1, \ldots, g_s)}{P_{\bar{l}}(\zeta_1^{l_1}, \ldots, \zeta_s^{l_s})}, \text{ where } 0 \leqslant l_i \leqslant n_i - 1 \text{ for } i = 1, \ldots, s. \tag{1.3}$$

PROOF. The image of $e_{\bar{l}}$ under the isomorfism $KG \simeq \oplus_{i=1}^{n} K$ is:

$$(\ldots, \frac{P_{\bar{l}}(\zeta_1^{k_1},\ldots,\zeta_s^{k_s})}{P_{\bar{l}}(\zeta_1^{l_1},\ldots,\zeta_s^{l_s})}, \ldots)_{0\leqslant k_i\leqslant n_i-1} = (0,\ldots,0, \frac{P_{\bar{l}}(\zeta_1^{l_1},\ldots,\zeta_s^{l_s})}{P_{\bar{l}}(\zeta_1^{l_1},\ldots,\zeta_s^{l_s}))}, 0,\ldots,0) = \tag{1.4}$$

$(0,\ldots,0,1,0,\ldots,0)$, with 1 in the position $(l_1,\ldots,l_s)$.

□

COROLLARY 1.2. *With the same notation as in Theorem 1.1 suppose that G is a cyclic group of order n generated by g. For ζ a primitive root of unity of order n, the primitive idempotents in KG are:*

$$e_l := \frac{\zeta^{n-l}}{n} \prod_{i=0,i\neq l}^{n-1} (g-\zeta^i), \text{ where } 0 \leqslant l \leqslant n-1. \tag{1.5}$$

PROOF. From Theorem 1.1, it follows that:

$$e_l = \prod_{i=0,i\neq l}^{n-1} \frac{(g-\zeta^i)}{(\zeta^l-\zeta^i)} = \frac{\zeta^l}{n} \prod_{i=0,i\neq l}^{n-1} (g-\zeta^i). \tag{1.6}$$

□

When the field is not algebraically closed, we have the following situation:

THEOREM 1.3. *Let $G \simeq C_1 \times \cdots \times C_s$ be a finite abelian group of order n, with $C_i = \langle g_i; g_i^{n_i} = 1\rangle$ the cyclic group of order n_i generated by g_i, and let K be a field such that $char(K)$ does not divide n. Define $m := lcm(n_1,\ldots,n_s)$. Consider $\mathcal{A} := Aut(K(\zeta_m)|K)$, the Galois group of the field extension $K(\zeta_m)|K$. Then, for a fixed s-tuple of integers $\bar{l} = (l_1,\ldots,l_s)$, with $0 \leqslant l_i \leqslant n_i - 1$, the element $e_{\bar{l}}$ defined below is a primitive idempotent of the abelian group algebra KG:*

$$e_{\bar{l}} := \sum_{\sigma\in\mathcal{A}} \frac{P_{\bar{l}}^{\sigma}(g_1,\ldots,g_s)}{\sigma(P_{\bar{l}}(\zeta_{n_1}^{l_1},\ldots,\zeta_{n_s}^{l_s}))},$$

the sum of all galois conjugates of $P_{\bar{l}}(g_1,\ldots,g_s)/P_{\bar{l}}(\zeta_{n_1}^{l_1},\ldots,\zeta_{n_s}^{l_s})$, where $P_{\bar{l}}^{\sigma}$ denotes the polynomial in $K[X_1,\ldots,X_s]$ obtained by applying σ to the coefficients of $P_{\bar{l}}$. Furthermore, these are all the primitive idempotents of KG.

PROOF. From the Theorem 1.1, it follows that

$$f_{\bar{l}} := \frac{P_{\bar{l}}(g_1,\ldots,g_s)}{P_{\bar{l}}(\zeta_1^{l_1},\ldots,\zeta_s^{l_s})} \tag{1.7}$$

is a primitive idempotent of $\overline{K}G$, where $\overline{K}$ is an algebraic closure of K. Therefore, $f_{\bar{l}}$ is also a primitive idempotent in $K(\zeta_m)G$. Notice that $K(\zeta_m)$ is the minimal field extension of K such that $f_{\bar{l}}$ belongs to $K(\zeta_m)G$. Each $\sigma \in \mathcal{A}$ induces a unique automorphism σ^* of $K(\zeta_m)G$, and thus, $\sigma^*(f_{\bar{l}})$ is still a primitive idempotent of $K(\zeta_m)G$ and of $\overline{K}G$. The elements $\sigma_1^*(f_{\bar{l}})$ and $\sigma_2^*(f_{\bar{l}})$ are distinct if $\sigma_1^* \neq \sigma_2^*$ and, since they are both primitive idempotents, they are orthogonal. Thus, $e_{\bar{l}}$, being a sum of orthogonal idempotents, is an idempotent. Clearly, $e_{\bar{l}} \in KG$, since $e_{\bar{l}}$ is the trace in the field extension $K(\zeta_m)|K$ of $f_{\overline{l}}$. We have yet to see that $e_{\overline{l}}$ is primitive; we will do so in the end of the proof.

Let e be a primitive idempotent in KG, then we may write $e = f_1 + \cdots + f_t$ where f_i are primitive idempotents in $\overline{K}G$. Let K_2 be the minimal field such that f_i belongs to K_2G for each $i = 1, \ldots, t$. Take $\tau \in Aut(K_2|K)$. Applying τ^* to e, we get $e = \tau^*(e) = \tau^*(f_1) + \cdots + \tau^*(f_t)$ and, by the unique representation in K_2G, we have that e is exactly the sum of the distinct galois conjugates of a primitive idempotent in K_2G. Therefore, the $e_{\overline{l}}$ defined as above are all the primitive idempotents of KG.

Now, let us see that $e_{\overline{l}}$ is primitive in KG. Suppose that we may write $e_{\overline{l}} = e_1 + \ldots + e_t$, with e_i nonzero primitive orthogonal idempotents in KG. The first part of the proof implies that, for all i, $e_i = \sum_{\sigma\in\mathcal{A}} \sigma(f_i)$, with f_i distinct primitive idempotents in $\overline{K}G$. So we have that $e_{\overline{l}} = \sum_{\sigma\in\mathcal{A}} \sigma(f_{\overline{l}}) = \sum_{i=1}^{t} \sum_{\sigma\in\mathcal{A}} \sigma(f_i)$. By the unique representation in KG, it follows that all the f_i's are Galois-conjugates of one another. But then we would have that the e_i's are all equal, contradiction. □

Corollary 1.4. *With the same notation as in Theorem 1.3, suppose that* $G = \langle g; g^n = 1\rangle$ *is the cyclic group of order* n *generated by* g. *Let* ζ *be a primitive root of unity of order* n. *Fix* $0 \leqslant l \leqslant n-1$. *Consider* $\mathcal{A} := Aut(K(\zeta)|K)$, *the Galois group of the field extension* $K(\zeta^l)|K$. *Then the element* e_l *defined below is a primitive idempotent of the abelian group algebra* KG*:*

$$e_l := \sum_{\sigma\in\mathcal{A}} \sigma\left(\frac{\zeta^{n-l}}{n}\right) \prod_{i=0, i\neq l}^{n-1} (g - \sigma(\zeta^i)),$$

the sum of all distinct galois conjugates of $\zeta^{n-l}/n \prod_{i=0,i\neq l}^{n-1} (g - \zeta^i)$. *Furthermore, these are all the primitive idempotents of* KG.

Remark 1.5. Determining a cyclic code over a finite field corresponds to determine an ideal of a group algebra KG, where G is a cyclic group. In this case, all the ideals of KG are direct summands $\oplus_{s=1}^{l\leqslant n} KGe_{i_s}$, where $e_1, \ldots, e_n$ are the primitive idempotents in KG determined in Corollary 1.4.

2. The nilpotent case

For a subset H of G such that $char(K)$ does not divide $|H|$, we define the element $\widehat{H}$ of KG as:

$$\widehat{H} = \frac{1}{|H|} \sum_{h\in H} h. \tag{2.1}$$

If H is a subgroup of G, then $\widehat{H}$ is an idempotent of KG, and it is central in KG iff H is a normal subgroup of G.

For e a primitive central idempotent of KG, let $G_e = \{g \in G; eg = e\}$. Clearly, G_e is a normal subgroup of G and $e\widehat{G_e} = e$, thus e is also a primitive central idempotent of $(KG)\widehat{G_e} \simeq K(G/G_e)$, and the image $\overline{e}$ of e in $K(G/G_e)$, is a primitive central idempotent of $K(G/G_e)$.

Also notice that, if N is a normal subgroup of G contained in G_e, then, clearly $(G/N)_{\overline{e}} = G_e/N$, where $\overline{e}$ denotes the image of e in $K(G/N)$.

REMARK 2.1. Let G be a metabelian group and A an abelian normal subgroup of G such that G/A is abelian. Then the primitive central idempotents e of KG having $G_e = A$ are given by $e = \widehat{A}f$, where $f \in KG$ is such that $\overline{f}$ is a primitive idempotent in $K(G/A)$, having $(G/A)_{\overline{f}} = 1$. Recall that Theorem 1.3 yields all primitive idempotents of $K(G/A)$.

We denote by $\mathcal{Z}_2(G)$ the second center of G, which is the unique subgroup of G such that $\frac{\mathcal{Z}_2(G)}{\mathcal{Z}(G)}$ is the center of $\frac{G}{\mathcal{Z}(G)}$.

We need a result from Jespers-Leal-Paques ([**1**], Prop. 2.1). This result is stated in the reference for the group algebra $\mathbb{Q}G$, where G is a nilpotent group. We observe that the proof given is still valid for the case KG, when K is a field such that $char(K)$ does not divide $|G|$. In this context, the result is:

PROPOSITION 2.2. *Let G be a finite nilpotent group, K be a field such that $char(K)$ does not divide $|G|$, $e \in KG$ and $G_1 = \mathcal{C}_G(\mathcal{Z}_2(G))$, the centralizer in G of the second center of G. Then e is a primitive central idempotent of KG with G_e trivial if and only if e is the sum of all G-conjugates of e_1, a primitive central idempotent of KG_1 satisfying $\bigcap_{g\in G}((G_1)_{e_1})^g = \{1\}$.*

PROOF. Suppose $e \in KG$ is a primitive central idempotent with $G_e = \{1\}$. Write $e = \sum_{g\in G}\alpha_g\widehat{\mathcal{C}}_g$, with each $\alpha_g \in K$. Because of [**1**, Lemma 2.3], for any $g \in G$ with $g \notin \mathcal{C}_G(\mathcal{Z}_2)$ there exists a non-trivial central element $w_g \in G$ of prime order such that $\widehat{\mathcal{C}}_g = \widehat{\mathcal{C}}_g\ \widehat{\langle w_g\rangle}$. Hence

$$e = \sum_{g\in\mathcal{C}_G(\mathcal{Z}_2)}\alpha_g\widehat{\mathcal{C}}_g + \sum_{g\notin\mathcal{C}_G(\mathcal{Z}_2)}\alpha_g\widehat{\mathcal{C}}_g\ \widehat{\langle w_g\rangle}.$$

Because $G_e = \{1\}$, [**1**, Lemma 2.1], yields that $e = e\,\varepsilon(G)$. As $\varepsilon(G)\widehat{\langle w_g\rangle} = 0$ we thus get that

$$e = e\varepsilon(G) = \sum_{g\in\mathcal{C}_G(\mathcal{Z}_2)}\alpha_g\widehat{\mathcal{C}}_g\cdot\varepsilon(G).$$

So we have shown that *supp* $(e) \subseteq G_1 = \mathcal{C}_G(\mathcal{Z}_2(G))$. Note that e is not necessarily a primitive central idempotent of KG_1. However, using standard arguments we get that

$$e = e_1^{g_1} + \cdots + e_1^{g_n},$$

the sum of all G-conjugates of a primitive central idempotent $e_1 \in KG_1$. Clearly $((G_1)_{e_1})^{g_i} = (G_1)_{e_1^{g_i}}$. Hence it easily follows that $\cap_{i=1}^n((G_1)_{e_1})^{g_i} = G_e = \{1\}$. This proves the necessity of the conditions.

Conversely, assume G is a finite nilpotent group and suppose e_1 is a primitive central idempotent of KG_1 with $G_1 = \mathcal{C}_G(\mathcal{Z}_2(G))$ and assume $\cap_{g\in G}((G_1)_{e_1})^g = \{1\}$. Let $e = e_1^{g_1} + \cdots + e_1^{g_n}$ be the sum of all G-conjugates of e_1. Clearly e is a central idempotent of KG and $G_e = \{1\}$. Write $e = f_1 + \cdots + f_k$, a sum of primitive central idempotents of KG. Note that for any non-trivial central subroup N of G, either $\widehat{N}e_1 = 0$ or $\widehat{N}e_1 = e_1$. However the latter is impossible as it implies $N \subseteq (G_1)_{e_1}$ and thus $N \subseteq \cap_{g\in G}((G_1)_{e_1})^g = \{1\}$. So we get that $\widehat{N}e_1 = 0$ and thus $\varepsilon(G)e_1 = e_1$. Consequently, $\varepsilon(G)e = e$ and thus $\varepsilon(G)f_1 = f_1$. Therefore $G_{f_1} = \{1\}$ and thus by the first part of the proof, $f_1 \in KG_1$. Hence $e = f_1$ is a primitive central idempotent of KG. □

The following theorem yields an explicit formula for the primitive central idempotents in KG.

THEOREM 2.3. *Let G be a finite nilpotent group and K be a field such that $char(K)$ does not divide $|G|$. The primitive central idempotents of KG are precisely all elements of the form*

$$\sum_{g\in G}(e\widehat{H_m})^g, \tag{2.2}$$

the sum of all distinct G-conjugates of e, where e is an element of KG_m and $\overline{e}$ (the image of e in $K(G_m/H_m)$) is a primitive central idempotent in $K(G_m/H_m)$, having $(G_m/H_m)_{\overline{e}} = \{1\}$. The groups H_m and G_m are subgroups of G satisfying the following properties:

1. *$H_0 \subseteq H_1 \subseteq \ldots \subseteq H_m \subseteq G_m \subseteq \ldots \subseteq G_1 \subseteq G_0 = G$,*
2. *for $0 \leqslant i \leqslant m$, H_i is a normal subgroup of G_i, G_i/H_i is not abelian for $0 \leqslant i < m$, and G_m/H_m is abelian,*
3. *for $0 \leqslant i \leqslant m-1$, $G_{i+1}/H_i = \mathcal{C}_{G_i/H_i}(\mathcal{Z}_2(G_i/H_i))$,*
4. *for $1 \leqslant i \leqslant m$, $\bigcap_{x\in G_{i-1}/H_{i-1}} H_i^x = H_{i-1}$.*

PROOF. Let us first show that the element defined above, satisfying the listed conditions, is in fact a primitive central idempotent of KG. By condition 2, G_m/H_m is an abelian group, let $\overline{f_m} := \overline{e}$ the primitive central idempotent in $K(G_m/H_m)$, having $(G_m/H_m)_{\overline{e}} = \{1\}$. Since $K(G_m/H_m) \simeq (KG_m)\widehat{H_m}$, we have that $f_m := e\widehat{H_m}$ is a primitive central idempotent of $(KG_m)\widehat{H_m}$ and, thus, it is also a primitive central idempotent of KG_m. From $(G_m/H_m)_{\overline{e}} = \{1\}$ we have that $(G_m)_{f_m} = H_m$; so $(G_m/H_{m-1})_{\overline{f_m}} = H_m/H_{m-1}$. Define $\overline{f_{m-1}} := \sum_{\overline{g}\,\in\, G_{m-1}/H_{m-1}} \overline{f_m}^{\overline{g}}$ as the sum of all G_{m-1}/H_{m-1} conjugates of $\overline{f_m}$, then it is a central idempotent of $K(G_{m-1}/H_{m-1})$. From condition 4,

$$\bigcap_{g\in G_{m-1}/H_{m-1}} \Big((G_m/H_{m-1})_{\overline{f_m}}\Big)^g = \bigcap_{g\in G_{m-1}/H_{m-1}} (H_m/H_{m-1})^g = 1. \tag{2.3}$$

Conditions 3 and 4 provide the hypotheses for the Proposition 2.2, which yields that $\overline{f_{m-1}}$ is primitive in $K(G_{m-1}/H_{m-1}) \simeq (KG_{m-1})\widehat{H_{m-1}}$ and thus, it is also primitive central idempotent of KG_{m-1}. We also have, by condition 2, that:

$$\begin{aligned} f_{m-1} &= (\textstyle\sum_{g\in G_{m-1}} f_m^g)\widehat{H_{m-1}} = \sum_{g\in G_{m-1}}(f_m\widehat{H_{m-1}})^g = \\ &\textstyle\sum_{g\in G_{m-1}}(e\widehat{H_m}\widehat{H_{m-1}})^g = \sum_{g\in G_{m-1}}(e\widehat{H_m})^g, \end{aligned} \tag{2.4}$$

By induction, we obtain that $f_0 = \sum_{g\in G}(e\widehat{H_m})^g$ is a primitive central idempotent of KG.

Now, let $f_0 := d$ be a primitive central idempotent of KG, where G is a finite nilpotent group with nilpotency class c. Then $H_0 := G_d$ is a normal subgroup of

$G_0 := G$ and, since $f_0\widehat{H_0} = f_0$, we have that f_0 is a primitive central idempotent of $(KG)\widehat{H_0}$. From $(KG_0)\widehat{H_0} \simeq K(G_0/H_0)$, we get that $\overline{f_0}$, the image of f_0 in $K(G_0/H_0)$, is a a primitive central idempotent of $K(G_0/H_0)$. Clearly, $(G_0/H_0)_{\overline{f_0}} = \{1\}$.

If G_0/H_0 is an abelian group, we know $\overline{f_0}$ from Theorem 1.3 and $d = f_0 = \sum_{g\in G}(f_0\widehat{H_0})^g$, because $f_0\widehat{H_0} = f_0$ and f_0 is central.

If G_0/H_0 is not an abelian group, let G_1 be a subgroup of G_0 so that $G_1/H_0 = \mathcal{C}_{G_0/H_0}(\mathcal{Z}_2(G_0/H_0)) \neq G_0/H_0$. Then by Proposition 2.2,we get:

$$\overline{f_0} = \sum_{\overline{g}\in G_0/H_0} \overline{f_1}^{\overline{g}}, \tag{2.5}$$

where $\overline{f_1}$ is a primitive central idempotent of $K(G_1/H_0)$, $\bigcap_{x\in G_0/H_0}(H_1/H_0)^x = \{1\}$, H_1 is the subgroup of G_0 containing H_0 so that $H_1/H_0 = (G_1/H_0)_{\overline{f_1}}$. So H_1 is a normal subgroup of G_1. Notice that, from the definition of G_1/H_0, its nilpotency class is at most $c-1$. Since $K(G_1/H_1) \simeq (K(G_1/H_0)(\widehat{H_1/H_0})$, we have that $\overline{f_1}$, the image of $\overline{f_1}$ in $K(G_1/H_1)$, is a primitive central idempotent of $K(G_1/H_1)$, having $(G_1/H_1)_{\overline{f_1}} = \{1\}$. If G_1/H_1 is an abelian group, then we know $\overline{f_1}$ from Theorem 1.3. If G_1/H_1 is not an abelian group the result follows by induction on the nilpotency class c of G. □

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