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Images of certain special functions pertaining to multiple Erd$\acute{e}$lyi-Kober operator of Weyl type

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Abstract. The aim in this paper is to establish the images of the product of certain special functions with $zt^h(t^\mu + c^\mu)^{-\rho}$ as an argument pertaining to the multiple Erdélyi-Kober operator due to Galué et al.

The results encompass several cases of interest for Riemann-Liouville operators, Erdélyi-Kober operator and Saigo operators etc. involving the product of certain special function of general argument.

1. Introduction and definitions : The multiple Erdélyi-Kober operator of Weyl type, introduced by Galu et al. [10], is defined as:

$$K_{(\tau_w),\,(\lambda_w),\,r}^{(\eta_w),\,(\zeta_w)} f(x) = \begin{cases} \int\limits_1^\infty H_{r,\,r}^{r,\,0}\left[\frac{1}{y}\left|\begin{array}{l}(\eta_w+\zeta_w+1/\tau_w,\ 1/\tau_w)_1^r\\(\eta_w+1/\lambda_w,\,1/\lambda_w)_1^r\end{array}\right.\right] f\,(xy)\ dy,\ if\ \sum\limits_1^r \zeta_w\ >\ 0 \\ f\,(x)\,,\,if\ \zeta_w=0,\,\lambda_w=\tau_w,\,w=1,\,2,\,...,\,r\end{cases} \tag{1.1}$$

where $\sum\limits_{w=1}^{r}\frac{1}{\lambda_w}\ \geqslant\ \sum\limits_{w=1}^{r}\frac{1}{\tau_w}$ and $f(x)\in C_{\dot\beta}^*$
The class $C_{\dot\beta}^*$ is defined in the form [10, p.56].

$$C_\beta^* = \{f\,(x) = x^q\,\overset{\approx}{f}(x); q<\beta^*, \overset{\approx}{f}\in C(0,\infty), |\,\overset{\approx}{f}(x)|<A_{\overset{\approx}{f}}\} \tag{1.2}$$

and $\beta^* \leqslant \max(\lambda_w, \eta_w)$
Galué et al. [10, p.56] represented that

$$K_{(\tau_w),\,(\lambda_w),\,r}^{(\eta_w),\,(\zeta_w)}\,x^\rho\ =\ \prod_{w=1}^{r}\frac{\Gamma\,(\eta_w-\rho/\lambda_w)}{\Gamma\,(\eta_w+\zeta_w-\rho/\lambda_w)}\,x^\rho \tag{1.3}$$

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In the form of Pochhammer symbol $(a)_{n_1}$, defined as

$$\begin{aligned} (a)_{n_1} &= \tfrac{\Gamma(a+n_1)}{\Gamma(a)} \\ &= \begin{cases} 1, & if\ n_1 = 0, \\ a\,(a+1)\ldots(a+n_1-1)\,, & \forall\ n_1 \in N \end{cases} \end{aligned} \tag{1.4}$$

we can write

$$(1-x)^{-\alpha} = \sum_{n_1=0}^{\infty} \frac{(\alpha)_{n_1}}{n_1!}\, x^{n_1} \tag{1.5}$$

A general class of multivariable polynomials of Srivastava and Garg [8] is defined and represented in the following form

$$S_n^{w_1,\,\ldots,\,w_s}\,[x_1,\ldots,x_s] \;=\; \sum_{k_1,\,\ldots,\,k_s\,=0}^{w_1k_1+\,\ldots\,+\,w_sk_s\,\leqslant\,n} (-n)_{w_1k_1+\,\ldots\,+\,w_sk_s} A(n;k_1,\cdots,k_s)\frac{{x_1}^{k_1}}{k_1!},\ \ldots,\ \frac{{x_s}^{k_s}}{k_s!}, \tag{1.6}$$

$n, w_1, \ldots, w_s \in N_0 = \{0, 1, 2, \ldots\}$ and the coefficients $A_(n; k_1, \ldots, k_s), (k_j \in N_0; j = 1, \ldots, s)$ are arbitrary constants, real or complex.

For s = 1, the polynomials (1.6) reduces to a general class of polynomials due to Srivastava [1].

$$S_n^w[x] = \sum_{k=0}^{[n/w]} \frac{(-n)_{wk}}{k!}\ A_{n,k}x^k, n = 0, 1, 2, \ldots \tag{1.7}$$

where w is an arbitrary positive integer, the coefficients $A_n, k(n, k \in N_0)$ are arbitrary constants real or complex.

The following are the interesting special cases of this polynomials [7].

(i) Since

$$H_n[x] = \sum_{k=0}^{[n/2]} \frac{(-1)^k\ n!}{k!\ (n-2k)\,!}(2x)^{n-2k} \tag{1.8}$$

defines Hermite polynomials therefore in this case, if we take

$$w = 2, A_{n,k} = (\,-1)^k, S_n^2(x) \to\ x^{n/2} H_n\left(1/2\sqrt{x}\right) \tag{1.9}$$

(ii) On setting $w = 1, A_{n,k} = \begin{pmatrix} n+\alpha \\ n \end{pmatrix} \frac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k}$, S_n^1 reduces to the Jacobi polynomials $P_n^{(\alpha,\beta)}(1-2x)$, defined by Szegö [2, p. 68, eqn. (4.3.2)].

$$\begin{aligned} P_n^{(\alpha,\,\beta)}(x) &= \sum_{k=0}^{\infty} \begin{pmatrix} n+\alpha \\ n-k \end{pmatrix} \begin{pmatrix} n+\beta \\ k \end{pmatrix} \left(\tfrac{x-1}{2}\right)^k \left(\tfrac{x+1}{2}\right)^{n-k} \\ &\cdot \begin{pmatrix} n+\alpha \\ n \end{pmatrix} {}_2F_1\left[-n,\, \alpha+\beta+n+1;\, \alpha+1;\, \tfrac{1-x}{2}\right], \end{aligned} \tag{1.10}$$

The following series representation of the H-function given in [14] will be required in the proof.

$$H_{R,\,S}^{K,\,L}[z] = H_{R,\,S}^{K,\,L}\left[z \left| \begin{array}{c} (e_R,\, E_R) \\ (f_S,\, F_S) \end{array} \right.\right] = \sum_{h=1}^{L}\sum_{v_1=0}^{\infty} \frac{(-1)^{v_1}}{v_1!}\, \frac{\eta\ (\xi)}{E_h} \left(\frac{1}{z}\right)^{\xi}, \tag{1.11}$$

$$where\ \xi = \frac{e_h - 1 - v_1}{E_h}, and (h = 1, 2, \ldots, L) \tag{1.12}$$

and

$$\eta(\xi) = \frac{\prod_{j=1,\, j\neq h}^{L} \Gamma(1 - e_j - E_j\,\xi) \prod_{j=1}^{K} \Gamma(f_j + \xi F_j)}{\prod_{j=L+1}^{R} \Gamma(e_j + \xi E_j) \prod_{j=K+1}^{S} \Gamma(1 - f_j - \xi F_j)} \tag{1.13}$$

which exists for $z \neq 0$, if $\mu < 0$ and for $|z| > \beta 1$ if $\mu = 0$;

$$\mu = \sum_{j=1}^{S} F_j - \sum_{j=1}^{R} E_j\ and\ \beta = \prod_{j=1}^{R} (E_j)^{E_j} \prod_{j=1}^{S} (F_j)^{-F_j}$$

The multivariable H-function due to Srivastava and Panda [4] will be defined and represented in the following manner:

$$H[z_1, \ldots, z_n] = H\begin{matrix} 0, v : \left(u^{(1)}, v^{(1)}\right); \ldots; \left(u^{(N)}, v^{(N)}\right) \\ A, C : \left[B^{(1)}, D^{(1)}\right]; \ldots; \left[B^{(N)}, D^{(N)}\right] \end{matrix}$$
$$\left[\begin{matrix} z_1 \\ \vdots \\ z_N \end{matrix}\middle| \begin{matrix} \left[(a) : \theta^{(1)}, \ldots, \theta^{(N)}\right] : \left[b^{(1)}, \phi^{(1)}\right]; \ldots; \left[b^{(N)}, \phi^{(N)}\right] \\ \\ \left[(c) : \psi^{(1)}, \ldots, \psi^{(N)}\right] : \left[d^{(1)}, \delta^{(1)}\right]; \ldots; \left[d^{(N)}, \delta^{(N)}\right] \end{matrix}\right] \tag{1.14}$$
$$= \frac{1}{(2\pi i)^N} \int_{L_1} \ldots \int_{L_N} \prod_{i=1}^{N} \Omega(\xi_i)\ \chi_i(\xi_i)\, z_1^{\xi_1} \ldots z_N^{\xi_N}\, d\xi_1 \ldots d\xi_N,$$

where $i = (1)^{1/2}$

$$\Omega(\xi_i) = \frac{\prod_{j=1}^{u^{(i)}} \Gamma\left(d_j^{(i)} + \delta_j^{(i)}\xi_i\right) \prod_{j=1}^{v^{(i)}} \Gamma\left(1 - b_j^{(i)} - \phi_j^{(i)}\xi_i\right)}{\prod_{j=u^{(i)}+1}^{D^{(i)}} \Gamma\left(1 - d_j^{(i)} - \delta_j^{(i)}\xi_i\right) \prod_{j=v^{(i)}+1}^{B^{(i)}} \Gamma\left(b_j^{(i)} + \phi_j^{(i)}\xi_i\right)}, \forall\, (i = 1, 2, \ldots, N) \tag{1.15}$$

$$\chi_i(\xi_i) = \frac{\prod_{j=1}^{v} \Gamma\left(1 - a_j + \sum_{i=1}^{r} \theta_j^{(i)}\xi_i\right)}{\prod_{j=v+1}^{A} \Gamma\left(a_j + \sum_{i=1}^{r} \theta_j^{(i)}\xi_i\right) \prod_{j=1}^{C} \Gamma\left(1 - c_j - \sum_{i=1}^{r} \psi_j^{(i)}\xi_i\right)}, \tag{1.16}$$

and an empty product is interpreted as unity. A series representation of (1.15) is given by Olkha and Chausaria [16]. For the sake of brevity, and an empty product is interpreted as unity.

$$\alpha * = Re\left[t + (\mu\eta) \min_{1 \leqslant i' \leqslant L} \frac{f_{i'}}{F_{i'}} + (h_i + \mu\rho_i)\frac{b_j^{(i)}}{\varphi_j^{(i)}}\right] for\ 1 \leqslant j \leqslant w^{(i)},\ i \in N \tag{1.17}$$

2. Images Under Multiple Erdélyi-Kober Operator

Letting

$$\begin{aligned} f(x) &= x^{\rho}\,(x^{\mu}+c^{\mu})^{-\sigma}\; H\left[z_1 x^{-h_1}(x^{\mu}+c^{\mu})^{-\rho_1},\; \dots,\; z_N x^{-h_N}(x^{\mu}+c^{\mu})^{-\rho_N}\right] \\ &\quad .\; S_n^{w_1,\dots,w_s}\left[x^{P_1}\,(x^{\mu}+c^{\mu})^{-q_1},\; \dots,\; x^{P_s}(x^{\mu}+c^{\mu})^{-q_s}\right] \\ &\quad .\; H_{R,\,S}^{K,\,L}\left[z x^{t}\,(x^{\mu}+c^{\mu})^{-\eta}\;\middle|\begin{pmatrix} e_R, E_R \\ f_S, F_S \end{pmatrix}\right] \end{aligned} \tag{2.1}$$

with

$$Re\left[-\alpha^{*} + \min_{1\leqslant k\leqslant r}(\lambda_k\,\gamma_k)\right] > 0, \sum_{i=1}^{r}\frac{1}{\lambda_i} \geqslant \sum_{j=1}^{r}\frac{1}{\tau_i}\quad and$$

$\eta, \rho, \sigma, h_i, \rho_i, (i = 1, ..., N), p_i, q_i (i = 1, ..., s) > 0$ then there holds the following formula

$$K_{(\tau_w),\,(\lambda_w),\,r}^{(n_w),\,(\zeta_w)}\,[f(x)]$$

$$= x^{\rho}c^{-\mu\sigma}\sum_{k_1,\dots,k_s=0}^{w_1k_1+\dots+w_sk_s\leqslant n}(-n)_{w_1k_1+\dots+w_sk_s}\;A\;(n;k_1,\dots,k_S)\;\frac{c^{-\mu\sum_{i=1}^{s}q_ik_i}}{k_1!,\dots,k_s!}\;x^{\sum_{i=1}^{s}p_ik_i}\;\sum_{l=0}^{\infty}\frac{(-1)^l x^{\mu l}}{l!c^{\mu l}}$$

$$.\;H_{R,\,S}^{K,\,L}\left[z^{\xi}x^{t\xi}c^{-\mu\eta\xi}\right]\;H_{A+r+1,\,C+r+1}^{r+1,\,v}\begin{matrix}:\left(u^{(1)},\,v^{(1)}\right);\dots;\left(u^{(N)},\,v^{(N)}\right)\\ :\left[B^{(1)},\,D^{(1)}\right];\dots;\left[B^{(N)},\,D^{(N)}\right]\end{matrix}$$

$$\left[\begin{matrix}\frac{z_1}{x^{h_1}c^{\mu\rho_1}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{z_N}{x^{h_N}c^{\mu\rho_N}}\end{matrix}\;\middle|\;\begin{matrix}\left[(a):\theta^{(1)},\,\dots,\,\theta^{(N)}\right],\;\left[1-\Delta-l:\rho_1,\,\dots,\rho_N\right],\\ \left[1-\eta_j+E:\frac{h_1}{\lambda_j},\,\dots,\,\frac{h_N}{\lambda_j}\right]_1^r:\;\left[b^{(1)},\phi^{(1)}\right];\;\dots;\;\left[b^{(N)},\phi^{(N)}\right]\\ \left[(c):\psi^{(1)},\,\dots,\,\psi^{(N)}\right]\;:\;\left[d^{(1)},\delta^{(1)}\right];\dots;\;\left[d^{(N)},\delta^{(N)}\right],\\ \left[\Delta:\rho_1,\,\dots,\rho_N\right],\;\left[\eta_j+\zeta_j-E:\;\frac{h_1}{\lambda_j},\dots,\frac{h_N}{\lambda_j}\right]_1^r\end{matrix}\right],$$

$$K_{(\tau_w),\,(\lambda_w),\,r}^{(n_w),\,(\zeta_w)}\,[f(x)]$$

$$= x^{\rho}c^{-\mu\sigma}\sum_{k_1,\dots,k_s=0}^{w_1k_1+\dots+w_sk_s\leqslant n}(-n)_{w_1k_1+\dots+w_sk_s}\;A\;(n;k_1,\dots,k_S)\;\frac{c^{-\mu\sum_{i=1}^{s}q_ik_i}}{k_1!,\dots,k_s!}\;x^{\sum_{i=1}^{s}p_ik_i}\;\sum_{l=0}^{\infty}\frac{(-1)^l x^{\mu l}}{l!c^{\mu l}}$$

$$.\;H_{R,\,S}^{K,\,L}\left[z^{\xi}x^{t\xi}c^{-\mu\eta\xi}\right]\;H_{A+r+1,\,C+r+1}^{r+1,\,v}\begin{matrix}:\left(u^{(1)},\,v^{(1)}\right);\dots;\left(u^{(N)},\,v^{(N)}\right)\\ :\left[B^{(1)},\,D^{(1)}\right];\dots;\left[B^{(N)},\,D^{(N)}\right]\end{matrix}$$

$$\left[\begin{matrix}\frac{z_1}{x^{h_1}c^{\mu\rho_1}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{z_N}{x^{h_N}c^{\mu\rho_N}}\end{matrix}\;\middle|\;\begin{matrix}\left[(a):\theta^{(1)},\,\dots,\,\theta^{(N)}\right],\;\left[1-\Delta-l:\rho_1,\,\dots,\rho_N\right],\\ \left[1-\eta_j+E:\frac{h_1}{\lambda_j},\,\dots,\,\frac{h_N}{\lambda_j}\right]_1^r:\;\left[b^{(1)},\phi^{(1)}\right];\;\dots;\;\left[b^{(N)},\phi^{(N)}\right]\\ \left[(c):\psi^{(1)},\,\dots,\,\psi^{(N)}\right]\;:\;\left[d^{(1)},\delta^{(1)}\right];\dots;\;\left[d^{(N)},\delta^{(N)}\right],\\ \left[\Delta:\rho_1,\,\dots,\rho_N\right],\;\left[\eta_j+\zeta_j-E:\;\frac{h_1}{\lambda_j},\dots,\frac{h_N}{\lambda_j}\right]_1^r\end{matrix}\right],$$

(2.2)

$$where\ E = \frac{\left[\rho+\sum_{i=1}^{s}\rho_ik_i+\mu l-t\xi\right]}{\lambda_j},$$

$$\Delta = \sigma+\sum_{i=1}^{s}q_ik_i-\eta\xi$$

and the series in (2.2) is convergent.

Proof of 2.2 :
To establish (2.2), we express the multivariable H-function, general class of polynomials and H-function by using (1.14), (1.6) and (1.11) respectively. Then changing the order of integration and summations which is permissible under the conditions surrounding (2.2) and appealing to the result (1.3), we arrive at the desired result.

3. Applications
As an application of the result (2.2), we derive six interesting special cases. More special cases associated with various orthogonal polynomials and special functions can be derived by using the special cases of the polynomial $S_n^w[x]$ and the H-function of several variables.

(I) Taking s = 1 in (2.2), the polynomial (1.6) will reduce to and consequently, we obtain the following result.

$$K^{(\eta_w),\,(\zeta_w)}_{(\tau_w),\,(\lambda_w),\,r}\left[f_1(x)\right] = x^{\rho}c^{-\mu\sigma}\sum_{k=0}^{[n/w]}(-n)_{wk}\,\frac{A\,(n;k)\,x^{pk}}{k!}\,c^{-q\mu w}\sum_{l=o}^{\infty}\frac{(-1)^l x^{\mu l}}{l!\,c^{\mu l}}\,H^{r+1,\,v\;:\;(u^{(1)},\,v^{(1)});\,...;\,(u^{(N)},\,v^{(N)})}_{A+r+1,\,C+r+1\;:\;[B^{(1)},\,D^{(1)}];\,...;\,[B^{(N)},\,D^{(N)}]}\left[\begin{array}{c|l} \frac{z_1}{x^{h_1}c^{\mu\rho_1}} & \left[(a):\theta^{(1)},\,...,\,\theta^{(N)}\right],\,\left[1-\Delta^*-l:\ \rho_1,\,...,\rho_N\right], \\ \cdot & \left[1-\eta_\omega+E^*:\frac{h_1}{\lambda_\omega},\,...,\,\frac{h_N}{\lambda_\omega}\right]_1^r:\ \left[b^{(1)},\,\phi^{(1)}\right];\ ...;\ \left[b^{(N)},\,\phi^{(N)}\right] \\ \cdot & \left[(c):\psi^{(1)},\,...,\,\psi^{(N)}\right]\ :\ \left[d^{(1)},\delta^{(1)}\right];...;\ \left[d^{(N)},\delta^{(N)}\right], \\ \cdot & \\ \frac{z_N}{x^{h_N}c^{\mu\rho_N}} & \left[\Delta^*:\rho_1,\,...,\rho_N\right],\ \left[\eta_\omega+\zeta_\omega-E^*:\ \frac{h_1}{\lambda_\omega},\,...,\,\frac{h_N}{\lambda_\omega}\right]_1^r \end{array}\right] \cdot H^{K,\,L}_{R,\,S}\left[z^{\xi}x^{t\xi}c^{-\mu\eta\xi}\right] \tag{3.1}$$

$$where\ E^* = \frac{1}{\lambda_\omega}[\rho+pk+\mu l-t\xi],\ \Delta^* = (\sigma+qk-\eta\xi)$$

and

$$f_1(x) = x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma}H\left[z_1x^{-h_1}\,(x^{\mu}+c^{\mu})^{-\rho_1},\,...,\,z_Nx^{-h_N}\,(x^{\mu}+c^{\mu})^{-\rho_N}\right] \cdot S_n^w\left[x^p(x^{\mu}+c^{\mu})^{-q}\right]H^{K,\,L}_{R,\,S}\left[z^{\xi}x^{t\xi}c^{-\mu\eta\xi}\right]$$

(II) Setting s = 1, w = 2 and $A_{n,k} = (1)^k$ in (2.2), then by virtue of the result (1.9), we find that

$$K^{(\eta_w),\,(\zeta_w)}_{(\tau_w),\,(\lambda_w),\,r}\left[f_2(x)\right] = x^{\rho}c^{-\mu\sigma}\sum_{k=0}^{[n/2]}(-1)^k(-n)_{2k}\,\frac{c^{-2q\mu}\,x^{pk}}{k!}\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}\left(\frac{x}{c}\right)^{\mu l}\cdot H^{r+1,\,v\;:\;(u^{(1)},\,v^{(1)});\,...;\,(u^{(N)},\,v^{(N)})}_{A+r+1,\,C+r+1\;:\;[B^{(1)},\,D^{(1)}];\,...;\,[B^{(N)},\,D^{(N)}]}\left[\begin{array}{c|l} \frac{z_1}{x^{h_1}c^{\mu\rho_1}} & \left[(a):\theta^{(1)},\,...,\,\theta^{(N)}\right],\,\left[1-\Delta^*-l:\rho_1,\,...,\rho_N\right], \\ \cdot & \left[1-\eta_\omega+E^*:\frac{h_1}{\lambda_\omega},\,...,\,\frac{h_N}{\lambda_\omega}\right]_1^r:\ \left[b^{(1)},\,\phi^{(1)}\right];\ ...;\ \left[b^{(N)},\,\phi^{(N)}\right] \\ \cdot & \left[(c):\psi^{(1)},\,...,\,\psi^{(N)}\right]\ :\ \left[d^{(1)},\delta^{(1)}\right];...;\ \left[d^{(N)},\delta^{(N)}\right], \\ \cdot & \\ \frac{z_N}{x^{h_N}c^{\mu\rho_N}} & \left[\Delta^*:\rho_1,\,...,\rho_N\right],\ \left[\eta_\omega+\zeta_\omega-E^*;\ \frac{h_1}{\lambda_\omega},\,...,\,\frac{h_N}{\lambda_\omega}\right]_1^r \end{array}\right] \cdot H^{K,\,L}_{R,\,S}\left[z^{\xi}x^{t\xi}c^{-\mu\eta\xi}\right], \tag{3.2}$$

where E^* and Δ^* are defined in equation (3.1), the series in (3.2) is convergent and the conditions given with (2.2) are satisfied for s = 1 and

$$f_2(x) = x^{\rho+\frac{np}{2}}(x^{\mu}+c^{\mu})^{-\sigma-\frac{nq}{2}}.\ H\left[z_1x^{-h_1}\ (x^{\mu}+c^{\mu})^{-\rho_1},\ ...,\ z_Nx^{-h_N}\ (x^{\mu}+c^{\mu})^{-\rho_N}\right]$$
$$.\ H_n\left[\frac{(x^{\mu}+c^{\mu})^{q/2}}{2x^{p/2}}\right]\ H_{R,\ S}^{K,\ L}\ \left[z^{\xi}x^{t\xi}c^{-\mu\eta\xi}\right]$$

(III) Next, if we set s = 1, w = 1 and

$$A_{n,k} = \binom{n+\alpha}{n}\frac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k},$$

then by virtue of (1.10),$S_n^1(x)$ reduces to the Jacobi polynomials and consequently, it yields

$$K_{(\tau_w),\ (\lambda_w),\ r}^{(n_w),\ (\zeta_w)}\ [f_3(x)]$$
$$= x^{\rho}c^{-\mu\sigma}\sum_{k=0}^{n}(-n)_k\binom{n+\alpha}{n}\frac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k}\frac{c^{-\mu q}x^{pk}}{k!}$$
$$.\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}\frac{x^{\mu l}}{c^{\mu l}}\ H_{A+r+1,\ C+r+1\ :\ (B^1,\ D^1);\ \cdots;\ (B^{(N)},\ D^{(N)})}^{r+1,\ v\ :\ (u^1,\ v^1);\ \cdots;\ (u^{(N)},\ v^{(N)})}$$
$$\left[\begin{array}{c|l} \frac{z_1}{x^{h_1}c^{\mu\rho_1}} & \left[(a):\theta^{(1)},\ ...,\ \theta^{(N)}\right],\ \left[1-\Delta^*-l:\rho_1,\ ...,\rho_N\right], \\ \cdot & \left[1-\eta_\omega+E^*:\frac{h_1}{\lambda_\omega},\ ...,\ \frac{h_N}{\lambda_\omega}\right]_1^r:\ \left[b^{(1)},\phi^{(1)}\right];\ ...;\ \left[b^{(N)},\phi^{(N)}\right] \\ \cdot & \left[(c):\psi^{(1)},\ ...,\ \psi^{(N)}\right]:\ \left[d^{(1)},\delta^{(1)}\right];...;\ \left[d^{(N)},\delta^{(N)}\right], \\ \frac{z_N}{x^{h_N}c^{\mu\rho_N}} & \left[\Delta^*:\rho_1,\ ...,\rho_N\right],\ \left[\eta_\omega+\zeta_\omega-E^*:\ \frac{h_1}{\lambda_\omega},\ ...,\ \frac{h_N}{\lambda_\omega}\right]_1^r \end{array}\right], \tag{3.3}$$

where E^* and Δ^* are defined in (3.1), the series in (3.3) is convergent and the conditions given with (2.2) are satisfied with s = 1 and

$$f_3(x) = x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma}H\left[z_1x^{-h_1}\ (x^{\mu}+c^{\mu})^{-\rho_1},\ ...,\ z_Nx^{-h_N}\ (x^{\mu}+c^{\mu})^{-\rho_N}\right]$$
$$.P_n^{(\alpha,\beta)}[1-2x^p(x^{\rho}+c^{\rho})^{-q}]\ H_{R,\ S}^{K,\ L}\ \left[z^{\xi}x^{t\xi}c^{-\mu\eta\xi}\right]$$

(IV) A result recently obtained by Chaurasia and Gupta [11] follows as a particular case of our main result.
(V) Taking $h_i = \rho_i = 0$ $(i = 1, 2, ..., N)$ and s = 1, the result in (2.2) reduces to a known result in (2.2) reduces to a known result recently given by Saxena, Ram and Chandak in [13].
(VI) Letting $t \longrightarrow 0, \eta \longrightarrow 0$ in (2.2) we find a known result obtained by Saxena, Ram and Chandak [15].

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