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# On certain integral formulas involving the generalized Lommel-Wright function and Jacobi polynomial

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Abstract. The main aim of the present paper is to derive some integral formulas involving a product of the generalized Lommel-Wright function and the classical Jacobi polynomial. The results obtained in the present paper are expressed in terms of well-known Srivastava and Daoust functions. Besides evaluating some novel forms of finite integrals, several other interesting particular cases, hitherto scattered in the literature, are also determined in the present investigation.

## 1. Introduction

Generalized special functions have their vast applications in numerous branches of science and engineering. The Lommel-Wright, Lommel, Struve, and generalized Bessel functions are ubiquitous in scientific literature. They arise in integrals and derivatives during the formulation of several physical phenomena of Mechanics, Physics, Engineering, and Astronomy. Specifically, integrals related to various types of Bessel functions are useful in material science disciplines, such as radiomaterial science, plasma material science, neutron physical research, and so on. For some recent investigations on integral involving Bessel functions, we refer the work of Choi and Agarwal [**2**] and Choi et al. [**3**].

Besides the above-mentioned applications, obviously non-exhaustive, the investigations of generalized special functions and integrals involving products of such generalized special functions may provide a suitable and unified theory of special functions. Due to their significant role in theory and applications, several researchers have investigated numerous results pertaining to integrals involving product of generalized special functions (for more details, see [**6, 7, 8, 9, 10, 11, 12, 14, 16**]).

Motivated by the work of other co-researchers, in the present research work, we have proposed two theorems based on single and double finite integrals involving the product of the generalized Lommel-Wright function and the classical Jacobi polynomial with a suitably generalized argument. Several novel and classical results are the special cases of the results derived in the current work. Some interesting special cases

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are delivered in terms of corollaries in the paper.
The generalized Lommel-Wright function $J^{\psi,m}_{\omega,\nu}(z)$ is defined by Oteiza et al. and is represented as [**16**]:

$$J^{\psi,m}_{\omega,\nu}(z) = (z/2)^{(\omega+2\nu)} \sum_{n=0}^{\infty} \frac{(-1)^n(\frac{z}{2})^{2n}}{(\Gamma(\nu+n+1))^m \Gamma(\omega+n\psi+\nu+1)}, \tag{1.1}$$

where $z \in \mathbb{C}\backslash(-\infty, 0]$, $\psi > 0$, $m \in \mathbb{N}$, $\omega, \nu \in \mathbb{C}$.

Particularly, if we take $m = 1$ in (1.1), we arrive at the following generalized form of the classical Bessel function [**15**], as a special case of generalized Lommel-Wright function:

$$J^{\psi}_{\omega,\nu}(z) = J^{\psi,1}_{\omega,\nu}(z) = (z/2)^{(\omega+2\nu)} \sum_{n=0}^{\infty} \frac{(-1)^n(\frac{z}{2})^{2n}}{\Gamma(\nu+n+1)\Gamma(\omega+n\psi+\nu+1)}, \tag{1.2}$$

with $z \in \mathbb{C}\backslash(-\infty, 0]$, $\psi > 0$, $\omega, \nu \in \mathbb{C}$.

Now, On putting $m = 1$, $\psi = 1$ and $\nu = \frac{1}{2}$ in (1.1), we get the following relationship of the generalized Lommel-Wright function with the Struve function $H_\omega(z)$ (see [**23**], p.328):

$$H_\omega(z) = J^{1,1}_{\omega,1/2}(z) = (z/2)^{(\omega+1)} \sum_{n=0}^{\infty} \frac{(-1)^n(\frac{z}{2})^{2n}}{\Gamma(n+\frac{3}{2})\Gamma(\omega+n+\frac{3}{2})}, \tag{1.3}$$

where $z, \omega \in \mathbb{C}$.

Next, if we let $m = 1$, $\psi = 1$ and $\nu = 0$ in (1.1), we obtain the well-known Bessel function $J_\omega(z)$ [**23**] as follows:

$$J_\omega(z) = J^{1,1}_{\omega,0}(z) = \sum_{n=0}^{\infty} \frac{(-1)^n(\frac{z}{2})^{\omega+2n}}{\Gamma(\omega+n+1)n!}, \tag{1.4}$$

$z, \omega \in \mathbb{C}$, $z \neq 0$, $Re(\omega) > -1$.

The Jacobi polynomial $P^{(\alpha,\beta)}_n(z)$ is defined by (see [**18**], [**20**])

$$P^{(\alpha,\beta)}_n(z) = \frac{(1+\alpha)_n}{n!} {}_2F_1 \left[ \begin{matrix} -n, 1+\alpha+\beta+n; \\ 1+\alpha; \end{matrix} \frac{1-z}{2} \right], \tag{1.5}$$

which can also be rewritten as:

$$P^{(\alpha,\beta)}_n(z) = \sum_{k=0}^{n} \frac{(1+\alpha)_n(1+\alpha+\beta)_{n+k}}{k!(n-k)!(1+\alpha)_k(1+\alpha+\beta)_n} \left(\frac{z-1}{2}\right)^k, \tag{1.6}$$

where $(\gamma)_n$ is a Pochhammer symbol [**21**] defined by

$$(\gamma)_n = \frac{\Gamma(\gamma+n)}{\Gamma(\gamma)} = \begin{cases} 1 & \text{if } n = 0, \\ \gamma(\gamma+1)...(\gamma+n-1) & \text{if } n \in \mathbb{N}. \end{cases}$$

Using (1.5), and taking $z = 1$ we get the following interesting relationship

$$P^{(\alpha,\beta)}_n(1) = \frac{(1+\alpha)_n}{n!}. \tag{1.7}$$

For a particular choice of the parameters $\alpha$ and $\beta$ in (1.5) and (1.6) we can list some of the other interesting polynomials as the special cases of the Jacobi polynomial.

For $\alpha = \beta$, (1.6) reduces in to following well-known polynomial known as the Ultraspherical polynomial (see [**18**], [**20**]).

$$P_n^{(\alpha,\alpha)}(z) = \sum_{k=0}^{n} \frac{(1+\alpha)_n (1+2\alpha)_{n+k}}{k!(n-k)!(1+\alpha)_k (1+2\alpha)_n} \left(\frac{z-1}{2}\right)^k. \tag{1.8}$$

Also, for $\alpha = \beta = \mu - \frac{1}{2}$, we get from (1.6)

$$P_n^{(\mu-\frac{1}{2},\mu-\frac{1}{2})}(z) = \frac{(1+\mu)_n}{(2\mu)_n} C_n^{\mu}(z), \tag{1.9}$$

where $C_n^{\mu}(z)$ is the Gegenbauer polynomial (see [**18**], [**20**]).

Moreover, for $\alpha = \beta = -\frac{1}{2}$ and $\alpha = \beta = \frac{1}{2}$, (1.6) determine following relationships:

$$P_n^{(-\frac{1}{2},-\frac{1}{2})}(z) = \frac{(\frac{1}{2})_n}{(n)!} T_n(z), \tag{1.10}$$

and

$$P_n^{(\frac{1}{2},\frac{1}{2})}(z) = \frac{(\frac{3}{2})_n}{(n+1)!} U_n(z). \tag{1.11}$$

$T_n(z)$ and $U_n(z)$ involved in above (1.10) and (1.11) are the Chebyshev polynomials of first and second kind respectively (see [**18**], [**20**]).

Similarly, for $\alpha = \beta = 0$, (1.6) gives

$$P_n^{(0,0)}(z) = P_n(z), \tag{1.12}$$

where $P_n(z)$ is the Legendre polynomial (see [**18**], [**20**]).

Srivastava and Daoust [**19**] proposed a multivariate generalization of hypergeometric function, known as the Srivastava and Daoust function in the literature, is defined as:

$$F_{C:D^1;\dots;D^r}^{A:B^1;\dots;B^r}\left[\begin{matrix}(a:\theta^1,\dots,\theta^r):(b^1:\phi^1);\dots;(b^r:\phi^r); \\ (c:\varphi^1,\dots,\varphi^r):(d^1:\delta^1);\dots;(d^r:\delta^r);\end{matrix}\; x_1,\dots,x_r\right]$$

$$= \sum_{k_1,\dots,k_r=0}^{\infty} \frac{\Pi_{j=1}^{A}(a_j)_{k_1\theta_j^1+\dots+k_r\theta_j^{(r)}} \Pi_{j=1}^{B^1}(b_j^1)_{k_1\phi_j^1} \dots \Pi_{j=1}^{B^r}(b_j^{(r)})_{k_r\phi_j^{(r)}}}{\Pi_{j=1}^{C}(c_j)_{k_1\varphi_j^1+\dots+k_r\varphi_j^{(r)}} \Pi_{j=1}^{D^1}(d_j^1)_{k_1\delta_j^1} \dots \Pi_{j=1}^{D^r}(d_j^{(r)})_{k_r\delta_j^{(r)}}} \frac{x_1^{k_1}}{k_1!} \cdots \frac{x_r^{k_r}}{k_r!}. \tag{1.13}$$

In the above series form of the Srivastava and Daoust function the coefficients $\theta_j^{(l)}$, $j = 1,\dots,A$; $\phi_j^{(l)}$, $j = 1,\dots,B^{(l)}$; $\varphi_j^{(l)}$, $j = 1,\dots,C$; $\delta_j^{(l)}$, $j = 1,\dots,D^{(l)}$ are real and positive, and (a) abbreviates the array of A parameters $a_1,\dots,a_A$, $(b^l)$ abbreviates the array of $B^{(l)}$ parameters $b_j^{(l)}$, $j = 1,\dots,B^{(l)}$; $\forall\, l \in \{1,\dots,r\}$, with similar interpretations for $(c)$

and $(d^{(l)}), \forall\, l \in \{1, ..., r\}$ ; etcetra. For applications of Srivastava and Daoust function of Srivastava and Daoust [**22**], N.U. Khan et al. [**8**], N.U. Khan et al. [**9**], Pandey [**17**] and Chaurasia and Pandey [**1**].

Now, we consider two integrals with certain generalized arguments which are useful in deriving the main results of the current paper.

**Lemma 1.1** *If $Re(\zeta) > 0$, $Re(\eta) > 0$, $a_1 \neq a_2$, $\rho \in \mathbb{R}\backslash \{-\mu, -\lambda\}$ and the constant $\lambda$ and $\mu$ are such that the expression $[\rho(a_2 - a_1) + \lambda(t - a_1) + \mu(a_2 - t)]$, (where $a_1 \leqslant t \leqslant a_2$) is non-zero, then following result holds true:*

$$\int_{a_1}^{a_2} (t-a_1)^{\zeta-1}(a_2-t)^{\eta-1}[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}dt$$

$$= \frac{\Gamma(\zeta)\Gamma(\eta)}{(a_2-a_1)(\rho+\lambda)^{\zeta}(\rho+\mu)^{\eta}\Gamma(\zeta+\eta)}. \tag{1.14}$$

*The above integral is a generalization of the result earlier derived by MacRobert in 1961, for more details we refer* [**13**].

**Lemma 1.2** *If $Re(\zeta) > 0$, $Re(\eta) > 0$ and $a_1 \neq a_2$, where $a_1 \leqslant u \leqslant a_2$, $a_1 \leqslant v \leqslant a_2$, then there holds the following integral formula:*

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2} (u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta} dudv$$

$$= (a_2-a_1)^{2\zeta+\eta}\frac{\Gamma(\zeta)\Gamma(\eta)}{\Gamma(\zeta+\eta)}. \tag{1.15}$$

*The above integral is a generalized form of the result proposed by Edward in 1992, for more details we refer* [**4**].

## 2. Main Result

In this section, we derive two integral formulas involving the product of generalized Lommel-Wright function and Jacobi polynomial. The results are determined in terms of the well-known Srivastava and Daoust hypergeometric function.

**Theorem 2.1** *If $Re(\beta) > 0$, $Re(\alpha) > 0$, $Re(\eta) > 0$, $Re(\zeta) > 0$, $\psi > 0$, $\lambda_1, \lambda_2, (\lambda_1 + \lambda_2), m \in \mathbb{N}$, $\omega, \nu \in \mathbb{C}$, $\rho \in \mathbb{R}\backslash \{-\mu, -\lambda\}$ and $a_1 \neq a_2$ and the constant $\lambda$ and $\mu$ are such that the expression $[\rho(a_2 - a_1) + \lambda(t - a_1) + \mu(a_2 - t)]$ is non-zero, then for $a_1 \leqslant t \leqslant a_2$ there holds the following integral formula:*

$$\int_{a_1}^{a_2} (t-a_1)^{\zeta-1}(a_2-t)^{\eta-1}[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}$$

$$\times J_{\omega,\nu}^{\psi,m}\left[\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{(\lambda_1+\lambda_2)}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{(\lambda_1+\lambda_2)}}\right]dt$$

$$=\frac{\Gamma(\zeta+\omega\lambda_1+2\nu\lambda_1)}{(a_2-a_1)(\rho+\lambda)^{(\zeta+\omega\lambda_1+2\nu\lambda_1)}(\rho+\mu)^{(\eta+\omega\lambda_2+2\nu\lambda_2)}}$$

$$\times\frac{\Gamma(\eta+\omega\lambda_2+2\nu\lambda_2)}{\Gamma(\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2))(\Gamma(\nu+1))^m\Gamma(\omega+\nu+1)}$$

$$\times F^{2+\lambda_1+\lambda_2:0;0}_{1+(\lambda_1+\lambda_2)+m+\psi:0;1}\left[\begin{array}{l}[\Delta(\lambda_1;\zeta+\lambda_1\omega+2\nu\lambda_1):2,3],[\Delta(\lambda_2;\eta+\lambda_2\omega+2\nu\lambda_2):2,3],\\ \\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2)):2,3],\end{array}\right.$$

$$[1+\alpha+\beta:1,2],[1+\alpha:1,1]:\_;\_;$$

$$\underbrace{[\nu+1:1,1]}_{m\text{-times}},[\Delta(\psi;\omega+\nu+1):1,1],[1+\alpha+\beta:1,1]:\_;[1+\alpha;1];$$

$$\left.\frac{(-1)(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}\psi^\psi(\rho+\lambda)^{2\lambda_1}(\rho+\mu)^{2\lambda_2}},\frac{(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}\psi^\psi(\rho+\lambda)^{3\lambda_1}(\rho+\mu)^{3\lambda_2}}\right],\tag{2.1}$$

*where* $J^{\psi,m}_{\omega,\nu}(z)$ *and* $P_n^{(\alpha,\beta)}(z)$ *are known as the generalized Lommel-Wright function and Jacobi polynomial defined in (1.1) and (1.6) respectively, and* $\Delta(p;\tau)$ *abbreviates the arrangement of* $p$ *parameters* $\frac{\tau}{p},\frac{\tau+1}{p},...,\frac{\tau+p-1}{p}$ *and* $p\geqslant 1$.

**Proof:** To prove Theorem 2.1, we first express the generalized Lommel-Wright function and the Jacobi polynomial in the series form given by (1.1) and (1.6) respectively. Now, on interchanging the order of integration and summation, which is valid under the given conditions, we arrive at the following:

$$\int_{a_1}^{a_2}(t-a_1)^{\zeta-1}(a_2-t)^{\eta-1}[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}$$

$$\times\left(\frac{(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{\lambda_1+\lambda_2}}\right)^{(\omega+2\nu)}\sum_{n=0}^{\infty}\frac{(-1)^n(t-a_1)^{2\lambda_1 n}}{(\Gamma(\nu+n+1))^m}$$

$$\times\frac{(a_2-t)^{2\lambda_2 n}}{\Gamma(\omega+n\psi+\nu+1)[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{2n(\lambda_1+\lambda_2)}}\sum_{k=0}^{n}\frac{(1+\alpha)_n}{k!(n-k)!}$$

$$\times\frac{(-1)^k(1+\alpha+\beta)_{n+k}(t-a_1)^{k\lambda_1}(a_2-t)^{k\lambda_2}}{(1+\alpha)_k(1+\alpha+\beta)_n[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{k(\lambda_1+\lambda_2)}}dt$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(1+\alpha)_n(-1)^{n+k}(1+\alpha+\beta)_{n+k}}{k!(n-k)!(1+\alpha)_k(1+\alpha+\beta)_n(\Gamma(\nu+n+1))^m\Gamma(\omega+n\psi+\nu+1)}$$

$$\int_{a_1}^{a_2}(t-a_1)^{\zeta+2n\lambda_1+\omega\lambda_1+2\nu\lambda_1+k\lambda_1-1}(a_2-t)^{\eta+2n\lambda_2+\omega\lambda_2+2\nu\lambda_2+k\lambda_2-1}$$

$$[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta-2n(\lambda_1+\lambda_2)-\omega(\lambda_1+\lambda_2)-2\nu(\lambda_1+\lambda_2)-k(\lambda_1+\lambda_2)}dt.\tag{2.2}$$

Now, using Lemma 1.1, the RHS of (2.2) can be rewritten as:

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{n} \frac{(1+\alpha)_n(-1)^{n+k}(1+\alpha+\beta)_{n+k}}{k!(n-k)!(1+\alpha)_k(1+\alpha+\beta)_n(\Gamma(\nu+n+1))^m\Gamma(\omega+n\psi+\nu+1)}$$

$$\times \frac{\Gamma(\zeta+2n\lambda_1+\omega\lambda_1+2\nu\lambda_1+k\lambda_1)}{(a_2-a_1)(\rho+\lambda)^{(\zeta+2n\lambda_1+\omega\lambda_1+2\nu\lambda_1+k\lambda_1)}(\rho+\mu)^{(\eta+2n\lambda_2+\omega\lambda_2+2\nu\lambda_2+k\lambda_2)}}$$

$$\times \frac{\Gamma(\eta+2n\lambda_2+\omega\lambda_2+2\nu\lambda_2+k\lambda_2)}{\Gamma\left(\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2)+2n(\lambda_1+\lambda_2)+k(\lambda_1+\lambda_2)\right)}. \tag{2.3}$$

Moreover, using the following well-known identity ([**15**], p.57)

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n} B(k,n) = \sum_{n,k=0}^{\infty} B(k,n+k), \tag{2.4}$$

we write

$$= \frac{\Gamma(\zeta+\omega\lambda_1+2\nu\lambda_1)}{(a_2-a_1)(\rho+\lambda)^{(\zeta+\omega\lambda_1+2\nu\lambda_1)}(\rho+\mu)^{(\eta+\omega\lambda_2+2\nu\lambda_2)}}$$

$$\times \frac{\Gamma(\eta+\omega\lambda_2+2\nu\lambda_2)}{\Gamma(\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2))} \sum_{n,k=0}^{\infty} \frac{(-1)^{n+2k}}{k!n!(\Gamma(\nu+n+k+1))^m}$$

$$\times \frac{(1+\alpha)_{n+k}(1+\alpha+\beta)_{n+2k}(\zeta+\omega\lambda_1+2\nu\lambda_1)_{(2n\lambda_1+3k\lambda_1)}}{\Gamma(\omega+n\psi+k\psi+\nu+1)(1+\alpha)_k(1+\alpha+\beta)_{n+k}(\rho+\lambda)^{(2n\lambda_1+3k\lambda_1)}}$$

$$\times \frac{(\eta+\omega\lambda_2+2\nu\lambda_2)_{(2n\lambda_2+3k\lambda_2)}}{(\rho+\mu)^{(2n\lambda_2+3k\lambda_2)}(\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2))_{2n(\lambda_1+\lambda_2)+3k(\lambda_1+\lambda_2)}}, \tag{2.5}$$

as the RHS of (2.2). Further, the RHS of (2.2) can be represented in terms of Srivastava and Daoust function (1.13) which is the desired form given as the RHS of (2.1). This completes the proof of Theorem 2.1.

**Theorem 2.2** *If $Re(\beta)>0$, $Re(\alpha)>0$, $Re(\zeta)>0$, $Re(\eta)>0$, $\psi>0$, $\lambda_1,\lambda_2,(\lambda_1+\lambda_2), m\in\mathbb{N}$, $\omega,\nu\in\mathbb{C}$ and $a_2\neq a_1$, then for $a_1\leqslant u\leqslant a_2$, $a_1\leqslant v\leqslant a_2$ there holds the following integral formula:*

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2} (u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$

$$\times J_{\omega,\nu}^{\psi,m}\left[\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{\lambda_1+\lambda_2}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{\lambda_1+\lambda_2}}\right] dudv$$

$$= \frac{\Gamma(\zeta+\omega\lambda_1+2\nu\lambda_1)\Gamma(\eta+\omega\lambda_2+2\nu\lambda_2)(a_2-a_1)^{2\zeta+\eta+\omega(2\lambda_1+\lambda_2)+2\nu(2\lambda_1+\lambda_2)}}{\Gamma(\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2))(\Gamma(\nu+1))^m\Gamma(\omega+\nu+1)}$$

$$\times F^{2+\lambda_1+\lambda_2:0;0}_{1+(\lambda_1+\lambda_2)+m+\psi:0;1}\left[\begin{array}{l}[\Delta(\lambda_1;\zeta+\lambda_1\omega+2\nu\lambda_1):2,3],[\Delta(\lambda_2;\eta+\lambda_2\omega+2\nu\lambda_2):2,3],\\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2)):2,3],\end{array}\right.$$

$$[1+\alpha+\beta:1,2],[1+\alpha:1,1]:\_;\_;$$

$$\underbrace{[\nu+1:1,1]}_{m\text{-times}},[\Delta(\psi;\omega+\nu+1):1,1],[1+\alpha+\beta:1,1]:\_;[1+\alpha;1];$$

(2.6)

$$\left.\frac{(-1)(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}\psi^{\psi}(a_2-a_1)^{-2(2\lambda_1+\lambda_2)}},\frac{(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}\psi^{\psi}(a_2-a_1)^{-3(2\lambda_1+\lambda_2)}}\right],$$

*where $J^{\psi,m}_{\omega,\nu}(z)$ and $P^{(\alpha,\beta)}_n(z)$ are known as the generalized Lommel-Wright function and Jacobi polynomial defined in (1.1) and (1.6) respectively, and $\Delta(p;\tau)$ abbreviates the arrangement of $p$ parameters $\frac{\tau}{p},\frac{\tau+1}{p},...,\frac{\tau+p-1}{p}$ and $p\geqslant 1$.*

**Proof:** To prove Theorem 2.2, we first express the generalized Lommel-Wright function and Jacobi polynomial in series form, given in (1.1) and (1.6) respectively. Now, on interchanging the order of integration and summation, which is valid under the given conditions, we get

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$

$$\times\left(\frac{(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{\lambda_1+\lambda_2}}\right)^{(\omega+2\nu)}\sum_{n=0}^{\infty}\frac{(-1)^n(u-a_1)^{2n\lambda_1}(a_2-v)^{2n\lambda_1}}{(\Gamma(\nu+n+1))^m}$$

$$\times\frac{(a_2-u)^{2n\lambda_2}}{\Gamma(\omega+n\psi+\nu+1)\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{2n(\lambda_1+\lambda_2)}}\sum_{k=0}^{n}\frac{(1+\alpha)_n(-1)^k}{k!(n-k)!(1+\alpha)_k}$$

$$\times\frac{(1+\alpha+\beta)_{n+k}(u-a_1)^{\lambda_1 k}(a_2-v)^{\lambda_1 k}(a_2-u)^{\lambda_2 k}}{(1+\alpha+\beta)_n\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{k(\lambda_1+\lambda_2)}}dudv$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(1+\alpha)_n(-1)^{n+k}(1+\alpha+\beta)_{n+k}}{k!(n-k)!(1+\alpha)_k(1+\alpha+\beta)_n(\Gamma(\nu+n+1))^m\Gamma(\omega+n\psi+\nu+1)}$$

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta+\omega\lambda_1+2\nu\lambda_1+2n\lambda_1+\lambda_1 k}(a_2-v)^{\zeta+\omega\lambda_1+2\nu\lambda_1+2n\lambda_1+\lambda_1 k-1}$$

$$(a_2-u)^{\eta+\omega\lambda_2+2\nu\lambda_2+2n\lambda_2+\lambda_2 k-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]$$

$$^{1-\zeta-\eta-\omega\lambda_1-\omega\lambda_2-2\nu\lambda_1-2\nu\lambda_2-2n(\lambda_1+\lambda_2)-k(\lambda_1+\lambda_2)}dudv. \tag{2.7}$$

Using Lemma 1.2 and identity (2.4) in the RHS of the above (2.7), we get following expression as the RHS of (2.7).

$$=\frac{\Gamma(\zeta+\omega\lambda_1+2\nu\lambda_1)\Gamma(\eta+\omega\lambda_2+2\nu\lambda_2)(a_2-a_1)^{2\zeta+\eta+\omega(2\lambda_1+\lambda_2)+2\nu(2\lambda_1+\lambda_2)}}{\Gamma(\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2))}$$

$$\times\sum_{n,k=0}^{\infty}\frac{(1+\alpha)_{n+k}(1+\alpha+\beta)_{n+2k}(a_2-a_1)^{2n(2\lambda_1+\lambda_2)+3k(2\lambda_1+\lambda_2)}}{k!n!(1+\alpha)_k(1+\alpha+\beta)_{n+k}(\Gamma(\nu+n+k+1))^m\Gamma(\omega+n\psi+k\psi+\nu+1)}$$

$$\times\frac{(-1)^{n+2k}(\zeta+\omega\lambda_1+2\nu\lambda_1)_{(2n\lambda_1+3\lambda_1 k)}(\eta+\omega\lambda_2+2\nu\lambda_2)_{(2n\lambda_2+3\lambda_2 k)}}{(\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2))_{(2n(\lambda_1+\lambda_2)+3k(\lambda_1+\lambda_2))}}.$$

By applying the definition of the Srivastava and Daoust function (1.13), we arrive at the desired result. This completes the proof of Theorem 2.2.

## 3. Special cases

In this section, we deduce some corollaries of Theorem 2.1 and 2.2 and demonstrate some connections of the obtained results with the other novel and known results which are hitherto scattered in the literature.

**Corollary 3.1** *For $m=1$ and with all the other conditions stated in the theorem* 2.1*, the following integral formula involving the product of a generalized Lommel-Wright function (1.2) and Jacobi polynomial (1.6) holds true:*

$$\int_{a_1}^{a_2}(t-a_1)^{\zeta-1}(a_2-t)^{\eta-1}[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}$$

$$\times J_{\omega,\nu}^{\psi}\left[\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{\lambda_1+\lambda_2}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{\lambda_1+\lambda_2}}\right]dt$$

$$=\frac{\Gamma(\zeta+\omega\lambda_1+2\nu\lambda_1)}{(a_2-a_1)(\rho+\lambda)^{(\zeta+\omega\lambda_1+2\nu\lambda_1)}(\rho+\mu)^{(\eta+\omega\lambda_2+2\nu\lambda_2)}}$$

$$\times\frac{\Gamma(\eta+\omega\lambda_2+2\nu\lambda_2)}{\Gamma(\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2))\Gamma(\nu+1)\Gamma(\omega+\nu+1)}$$

$$\times F^{2+\lambda_1+\lambda_2:0;0}_{2+(\lambda_1+\lambda_2)+\psi:0;1}\left[\begin{array}{l}[\Delta(\lambda_1;\zeta+\lambda_1\omega+2\nu\lambda_1):2,3],[\Delta(\lambda_2;\eta+\lambda_2\omega+2\nu\lambda_2):2,3],\\ \\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2)):2,3],\end{array}\right.$$

$$[1+\alpha+\beta:1,2],[1+\alpha:1,1]:\_;\_;$$

$$[\nu+1:1,1],[\Delta(\psi;\omega+\nu+1):1,1],[1+\alpha+\beta:1,1]:\_;[1+\alpha;1];$$

$$\left.\frac{(-1)(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}\psi^{\psi}(\rho+\lambda)^{2\lambda_1}(\rho+\mu)^{2\lambda_2}},\frac{(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}\psi^{\psi}(\rho+\lambda)^{3\lambda_1}(\rho+\mu)^{3\lambda_2}}\right]. \tag{2.8}$$

**Corollary 3.2** *For $m=1,\ \psi=1,\nu=\frac{1}{2}$ and with all the other conditions stated in the theorem* 2.1*, the following integral formula involving the product of the Struve function (1.3) and Jacobi polynomial (1.6) holds true:*

$$\int_{a_1}^{a_2}(t-a_1)^{\zeta-1}(a_2-t)^{\eta-1}[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}$$
$$\times H_{\omega}\left[\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{\lambda_1+\lambda_2}}\right]$$
$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{\lambda_1+\lambda_2}}\right]dt$$
$$=\frac{\Gamma(\zeta+\omega\lambda_1+\lambda_1)}{(a_2-a_1)(\rho+\lambda)^{(\zeta+\omega\lambda_1+\lambda_1)}(\rho+\mu)^{(\eta+\omega\lambda_2+\lambda_2)}\Gamma\left(\zeta+\eta+\omega(\lambda_1+\lambda_2)+(\lambda_1+\lambda_2)\right)}$$
$$\times\frac{\Gamma(\eta+\omega\lambda_2+\lambda_2)}{\Gamma(\frac{3}{2})\Gamma(\omega+\frac{3}{2})}F^{2+\lambda_1+\lambda_2:0;0}_{3+(\lambda_1+\lambda_2):0;1}\left[\begin{matrix}[\Delta(\lambda_1;\zeta+\lambda_1\omega+\lambda_1):2,3],[\Delta(\lambda_2;\eta+\lambda_2\omega+\lambda_2):2,3],\\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta+\omega(\lambda_1+\lambda_2)+(\lambda_1+\lambda_2)):2,3],\end{matrix}\right.$$

$$[1+\alpha+\beta:1,2],[1+\alpha:1,1]:\_;\_;$$

$$[\omega+\tfrac{3}{2}:1,1],[\tfrac{3}{2}:1,1],[1+\alpha+\beta:1,1]:\_;[1+\alpha;1];$$

$$\left.\frac{(-1)(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(\rho+\lambda)^{2\lambda_1}(\rho+\mu)^{2\lambda_2}},\frac{(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(\rho+\lambda)^{3\lambda_1}(\rho+\mu)^{3\lambda_2}}\right]. \tag{2.9}$$

**Corollary 3.3** *For $m=1,\psi=1,\ \nu=0$ and with all the other conditions stated in the theorem* 2.1*, the following integral formula involving the product of a generalized Bessel function (1.4) and Jacobi polynomial (1.6) holds true:*

$$\int_{a_1}^{a_2}(t-a_1)^{\zeta-1}(a_2-t)^{\eta-1}[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{-\zeta-\eta}$$

$$\times J_{\omega}\left[\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{\lambda_1+\lambda_2}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(t-a_1)^{\lambda_1}(a_2-t)^{\lambda_2}}{[\rho(a_2-a_1)+\lambda(t-a_1)+\mu(a_2-t)]^{\lambda_1+\lambda_2}}\right]dt$$

$$= \frac{\Gamma(\zeta+\omega\lambda_1)\Gamma(\eta+\omega\lambda_2)}{(a_2-a_1)(\rho+\lambda)^{(\zeta+\omega\lambda_1)}(\rho+\mu)^{(\eta+\omega\lambda_2)}\Gamma(\zeta+\eta+\omega(\lambda_1+\lambda_2))\Gamma(\omega+1)}$$

$$\times F^{2+\lambda_1+\lambda_2:0;0}_{3+(\lambda_1+\lambda_2):0;1}\left[\begin{array}{l}[\Delta(\lambda_1;\zeta+\lambda_1\omega):2,3],[\Delta(\lambda_2;\eta+\lambda_2\omega):2,3],[1+\alpha+\beta:1,2],\\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta+\omega(\lambda_1+\lambda_2)):2,3],[1+\alpha+\beta:1,1],\end{array}\right.$$

$$\begin{array}{l}[1+\alpha:1,1]:\_;\_;\\ [\omega+1:1,1],[1:1,1]:\_;[1+\alpha;1];\end{array}\frac{(-1)(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(\rho+\lambda)^{2\lambda_1}(\rho+\mu)^{2\lambda_2}},$$

$$\left.\frac{(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(\rho+\lambda)^{3\lambda_1}(\rho+\mu)^{3\lambda_2}}\right]. \tag{2.10}$$

**Remark 3.4** *On taking $\rho=0$, $a_1=0, a_2=1, \lambda_1=1, \lambda_2=1$ Corollary 3.3 reduces into an interesting result derived by Khan et al. (see* [**10**]*, Theorem 2.2, p.5, Eq.(2.5)).*

**Corollary 3.5** *For $m=1$ and with all the other conditions stated in the theorem 2.2, the following integral formula involving the product of a generalized Lommel-Wright function (1.2) and Jacobi polynomial (1.6) holds true:*

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$

$$\times J^{\psi}_{\omega,\nu}\left[\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{\lambda_1+\lambda_2}}\right]$$

$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{\lambda_1+\lambda_2}}\right]dudv$$

$$= \frac{\Gamma(\zeta+\omega\lambda_1+2\nu\lambda_1)\Gamma(\eta+\omega\lambda_2+2\nu\lambda_2)(a_2-a_1)^{2\zeta+\eta+\omega(2\lambda_1+\lambda_2)+2\nu(2\lambda_1+\lambda_2)}}{\Gamma(\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2))\Gamma(\nu+1)\Gamma(\omega+\nu+1)}$$

$$\times F^{2+\lambda_1+\lambda_2:0;0}_{2+(\lambda_1+\lambda_2)+\psi:0;1}\left[\begin{array}{l}[\Delta(\lambda_1;\zeta+\lambda_1\omega+2\nu\lambda_1):2,3],[\Delta(\lambda_2;\eta+\lambda_2\omega+2\nu\lambda_2):2,3],\\ \\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta+\omega(\lambda_1+\lambda_2)+2\nu(\lambda_1+\lambda_2)):2,3],\end{array}\right.$$

$$[1+\alpha+\beta:1,2],[1+\alpha:1,1]:\_;\_;$$

$$[\nu+1:1,1],[\Delta(\psi;\omega+\nu+1):1,1],[1+\alpha+\beta:1,1]:\_;[1+\alpha;1];$$

(2.11)
$$\left.\frac{(-1)(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}\psi^{\psi}(a_2-a_1)^{-2(2\lambda_1+\lambda_2)}},\frac{(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}\psi^{\psi}(a_2-a_1)^{-3(2\lambda_1+\lambda_2)}}\right].$$

**Corollary 3.6** *For $m=1,\psi=1,\ \nu=\frac{1}{2}$ and with all the other conditions stated in the theorem 2.2, the following integral formula involving the product of a generalized Struve function (1.3) and Jacobi polynomial (1.6) holds true:*

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$
$$\times H_{\omega}\left[\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{\lambda_1+\lambda_2}}\right]$$
$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{\lambda_1+\lambda_2}}\right]dudv$$
$$=\frac{\Gamma(\zeta+\omega\lambda_1+\lambda_1)\Gamma(\eta+\omega\lambda_2+\lambda_2)(a_2-a_1)^{2\zeta+\eta+\omega(2\lambda_1+\lambda_2)+(2\lambda_1+\lambda_2)}}{\Gamma(\zeta+\eta+\omega(\lambda_1+\lambda_2)+(\lambda_1+\lambda_2))\Gamma(\frac{3}{2})\Gamma(\omega+\frac{3}{2})}$$
$$\times F^{2+\lambda_1+\lambda_2:0;0}_{3+(\lambda_1+\lambda_2):0;1}\left[\begin{array}{l}[\Delta(\lambda_1;\zeta+\lambda_1\omega+\lambda_1):2,3],[\Delta(\lambda_2;\eta+\lambda_2\omega+\lambda_2):2,3],\\ \\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta+\omega(\lambda_1+\lambda_2)+(\lambda_1+\lambda_2)):2,3],\end{array}\right.$$

$$[1+\alpha+\beta:1,2],[1+\alpha:1,1]:\_;\_;$$

$$[\omega+\tfrac{3}{2}:1,1],[\tfrac{3}{2}:1,1],[1+\alpha+\beta:1,1]:\_;[1+\alpha;1];$$

(2.12)
$$\left.\frac{(-1)(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(a_2-a_1)^{-2(2\lambda_1+\lambda_2)}},\frac{(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(a_2-a_1)^{-3(2\lambda_1+\lambda_2)}}\right].$$

**Corollary 3.7** *For $m=1, \psi=1, \nu=0$ and with all the other conditions stated in the theorem 2.2, the following integral formula involving the product of a generalized Bessel function (1.4) and Jacobi polynomial (1.6) holds true:*

$$\int_{a_1}^{a_2}\int_{a_1}^{a_2}(u-a_1)^{\zeta}(a_2-v)^{\zeta-1}(a_2-u)^{\eta-1}\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{1-\zeta-\eta}$$
$$\times J_{\omega}\left[\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{\lambda_1+\lambda_2}}\right]$$
$$\times P_n^{(\alpha,\beta)}\left[1-\frac{2(u-a_1)^{\lambda_1}(a_2-v)^{\lambda_1}(a_2-u)^{\lambda_2}}{\left[1-\frac{(u-a_1)(v-a_1)}{(a_2-a_1)^2}\right]^{\lambda_1+\lambda_2}}\right]dudv$$
$$=\frac{\Gamma(\zeta+\omega\lambda_1)\Gamma(\eta+\omega\lambda_2)(a_2-a_1)^{2\zeta+\eta+\omega(2\lambda_1+\lambda_2)}}{\Gamma(\zeta+\eta+\omega(\lambda_1+\lambda_2))\Gamma(\omega+1)}$$
$$\times F^{2+\lambda_1+\lambda_2:0;0}_{3+(\lambda_1+\lambda_2):0;1}\left[\begin{matrix}[\Delta(\lambda_1;\zeta+\lambda_1\omega):2,3],[\Delta(\lambda_2;\eta+\lambda_2\omega):2,3],[1+\alpha+\beta:1,2],\\ [\Delta((\lambda_1+\lambda_2);\zeta+\eta+\omega(\lambda_1+\lambda_2)):2,3],[\omega+1:1,1],\end{matrix}\right.$$
$$\begin{matrix}[1+\alpha:1,1]:\_;\_;\\ [1+\alpha+\beta:1,1],[1:1,1]:\_;[1+\alpha;1];\end{matrix}$$

$$\left.\frac{(-1)(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(a_2-a_1)^{-2(2\lambda_1+\lambda_2)}},\frac{(\lambda_1^{\lambda_1})(\lambda_2^{\lambda_2})}{(\lambda_1+\lambda_2)^{(\lambda_1+\lambda_2)}(a_2-a_1)^{-3(2\lambda_1+\lambda_2)}}\right]. \tag{2.13}$$

**Remark 3.8** *For $a_1=0, a_2=1, \lambda_1=1, \lambda_2=1$ Corollary 3.7 reduces into an interesting result investigated by Ghayasuddin et al. (see* [**5**]*, Theorem 2.1, p.6, Eq.(18)).*

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