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On π-images of separable metric spaces

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Abstract. We prove that a space is a sequentially-quotient (pseudo-sequence-covering), π-image of a separable metric space if and only if it has a point-star network consisting of countable-cs^*-covers. We also investigate spaces with countable sn-networks.

1. Introduction

All spaces are assumed to be Hausdorff, and need not to be regular. π-mappings, were introduced by V. I. Ponomarev in [**14**], play an important role in generalized metric spaces theory ([**5**], [**7**], [**2**]). In recent years, π-mappings with some sequence-covering properties cause attention once again ([**16**], [**6**], [**9**]). We known that sequentially-quotient, compact images of (separable) metric spaces and pseudo-sequence-covering, compact images of (separable) metric spaces are equivalent ([**17**], [**11**]). However, if the analogous results on π-images is true. That is, we have the following question (also see [**9**, Question 3.1.14], for example).

Question 1.1. *Are sequentially-quotient, π-images of (separable) metric spaces and pseudo-sequence-covering, π-images of (separable) metric spaces equivalent?*

Taking this question into account, we prove that a space with a point-star network consisting of countable-cs^*-covers is a pseudo-sequence-covering, π-image of a separable metric space. As an application of this result, we give a positive answer to Question 1.1 for separable metric domains. In addition, we investigate spaces with countable sn-networks, prove that a sequentially-quotient, π-image of a separable metric space has countable sn-network, and the inversion is not true. But a space with a countable closed sn-network is a compact-covering, compact image of a separable metric space.

Throughout this paper, all mappings are continuous and onto. N denotes the set of all natural numbers. Let X be a space and let A be a subset of X. We call that a sequence $\{x_n : n \in N\} \bigcup \{x\}$ in X converging to x is eventually in A if $\{x_n : n >$

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$k\} \bigcup \{x\} \subset A$ for some $k \in N$. Let $\mathcal{P}$ be family of subsets of X. $\bigcup \mathcal{P}$ denotes the union $\bigcup \{P : P \in \mathcal{P}\}$. For $x \in X$, $(\mathcal{P})_x$ denotes the subfamily $\{P \in \mathcal{P} : x \in P\}$ of $\mathcal{P}$ and $st(x, \mathcal{P})$ denotes the union $\bigcup \{P \in \mathcal{P} : x \in P\}$. We call $\mathcal{P}$ is a network at some point $x \in X$ if whenever $x \in U$ with U open in X, then $x \in P \subset U$ for some $P \in (\mathcal{P})_x$. Let $f : X \longrightarrow Y$ be a mapping, and let $\mathcal{P}$ be a family of subsets of X. $f(\mathcal{P})$ denotes $\{f(P) : P \in \mathcal{P}\}$. We use the convention that every convergent sequence contains its limit point.

2. π-images of separable metric spaces

Definition 2.1. ([**14**]). Let (X, d) be a metric space, and let $f : X \longrightarrow Y$ be a mapping. f is called a π-mapping, if for every $y \in Y$ and for every neighborhood U of y in Y, $d(f^{-1}(y), X - f^{-1}(U)) > 0$.

Remark 2.1. Recall a mapping $f : X \longrightarrow Y$ is compact, if $f^{-1}(y)$ is a compact subset of X for every $y \in Y$. It is clear that every compact mapping from a metric space is a π-mapping.

Definition 2.2. Let $f : X \longrightarrow Y$ be a mapping.

(1) f is called a sequentially-quotient mapping([**1**]) if for every convergent sequence S of Y, there exists a convergent sequence L of X such that $f(L)$ is a subsequence of S.

(2) f is called a pseudo-sequence-covering mapping([**6**]) if for every convergent sequence S of Y, there exists a compact subset K of X such that $f(K) = S$.

(3) f is called a subsequence-covering mapping([**10**]) if for every convergent sequence S of Y, there exists a compact subset K of X such that $f(K)$ is a subsequence of S.

(4) f is called a compact-covering mapping([**13**]) if for every compact subset C of Y, there exists a compact subset K of X such that $f(K) = C$.

Remark 2.2. (1) Sequentially-quotient mapping (pseudo-sequence-covering mapping) $\Longrightarrow$ subsequence-covering mapping.

(2) Compact-covering mapping $\Longrightarrow$ pseudo-sequence-covering mapping $\Longrightarrow$(if the domain is metric) sequentially-quotient mapping.

Definition 2.3. ([**12**]). Let $\{\mathcal{P}_n : n \in N\}$ be a sequence of covers of a space X. $\{\mathcal{P}_n : n \in N\}$ is called a point-star network of X, if $\{st(x, \mathcal{P}_n) : n \in N\}$ is a network at x for every $x \in X$.

Definition 2.4. ([**9**]). Let $\mathcal{P}$ be a cover of a space X. $\mathcal{P}$ is called a cs^*-cover if for every convergent sequence S, there exist $P \in \mathcal{P}$ and a subsequence S' of S such that S' is eventually in P. $\mathcal{P}$ is called an fcs-cover if for every convergent sequence S converging to x, there exists a finite subfamily $\mathcal{F}$ of $(\mathcal{P})_x$ such that S is eventually in $\bigcup \mathcal{F}$. Furthermore $\mathcal{P}$ is called a countable-cs^*-cover (resp. countable-fcs-cover) if $\mathcal{P}$ is also countable.

It is easy to see that every fcs-cover of a space is a cs^*-cover and the inversion is not true. But every point-countable-cs^*-cover of a space is an fcs-cover by the following lemma.

LEMMA 2.1. *Let $\mathcal{P}$ be a point-countable cover of a space X. Then $\mathcal{P}$ is a cs^*-cover if and only if it is an fcs-cover.*

PROOF. The sufficiency is clear. We prove necessity. Let $\mathcal{P}$ be a point-countable-cs^*-cover of a space X. Let $S = \{x_n : n \in N\} \bigcup \{x\}$ be a sequence converging to $x \in X$. Put $(\mathcal{P})_x = \{P_n : n \in N\}$. We only need to prove that S is eventually in $\bigcup_{n \leqslant k} P_n$ for some $k \in N$. If for any $k \in N$, S is not eventually in $\bigcup_{n \leqslant k} P_n$, then for every $k \in N$, there exists $x_{n_k} \in S - \bigcup_{n \leqslant k} P_n$. We may assume $n_1 < n_2 < \cdots < n_{k-1} < n_k < n_{k+1} < \cdots$. Put $S' = \{x_{n_k} : k \in N\} \bigcup \{x\}$, then S' is a sequence converging to x. Since $\mathcal{P}$ is a cs^*-cover, there exist $m \in N$ and a subsequence S'' of S' such that S'' is eventually in P_m. Note that $P_m \in (\mathcal{P})_x$. This contradicts the construction of S'. □

Lin proved that every pseudo-sequence-covering mapping is sequentially-quotient if the domain is a space in which every point is G_δ([**9**]). We point out pseudo-sequence-covering mapping can be relaxed to subsequence-covering mapping. That is, we have the following lemma.

LEMMA 2.2. *Let $f : X \longrightarrow Y$ be a subsequence-covering mapping, and let every point in X be G_δ. Then f is a sequentially-quotient mapping.*

PROOF. Let S be a sequence in Y converging to $y \in Y$. Since f is subsequence-covering, there exists a compact subset K of X such that $f(K) = S'$ is a subsequence of S. Put $S' = \{y_n : n \in N\} \bigcup \{y\}$. Pick $x_n \in f^{-1}(y_n) \bigcap K$ for every $n \in N$, and put $L = \{x_n : n \in N\}$, then $L \subset K$. Notice that K is a compact subspace in which every point is G_δ. K is the first countable, so K is sequentially compact, thus there exists a subsequence $\{x_{n_k} : k \in N\}$ of L, which converges to some $x \in f^{-1}(y)$. This proves that f is sequentially-quotient. □

Now we give the main theorem in this paper.

THEOREM 2.1. *Let X be a space. Then the following are equivalent.*
(1) X is a pseudo-sequence-covering, π-image of a separable metric space.
(2) X is a subsequence-covering, π-image of a separable metric space.
(3) X is a sequentially-quotient, π-image of a separable metric space.
(4) X has a point-star network consisting of countable-cs^*-covers.
(5) X has a point-star network consisting of countable-fcs-covers.

PROOF. (1)$\Longrightarrow$(2) from Remark 2.2. (2)$\Longrightarrow$(3) from Lemma 2.2. (4)$\Longrightarrow$(5) from Lemma 2.1. We only need to prove that (3)$\Longrightarrow$(4) and (5)$\Longrightarrow$(1).

(3)$\Longrightarrow$(4): Let (M, d) be a separable metric space, and let $f : M \longrightarrow X$ be a sequentially-quotient, π-mapping. We write $B(a, \varepsilon) = \{b \in M : d(a, b) < \varepsilon\}$ for every $a \in M$, here $\varepsilon > 0$. Since M is separable, there exists a countable dense subset M' of M. For every $n \in N$, put $\mathcal{B}_n = \{B(a, 1/n) : a \in M'\}$, and put $\mathcal{P}_n = f(\mathcal{B}_n)$, then $\mathcal{P}_n$ is a countable cover of X.

Claim 1. $\{st(x, \mathcal{P}_n) : n \in N\}$ is a network at x for every $x \in X$.

Proof. Let $x \in U$ with U open in X. Since f is a π-mapping, there exists $n \in N$ such that $d(f^{-1}(x), M - f^{-1}(U)) > 1/n$. Pick $m \in N$ such that $m > 2n$. We prove that $st(x, \mathcal{P}_m) \subset U$ as follows. In fact, let $x \in f(B(a, 1/m)) \in \mathcal{P}_m$, here $a \in M'$.

Then $f^{-1}(x)\bigcap B(a,1/m)\neq\emptyset$. If $B(a,1/m)\not\subset f^{-1}(U)$, then $d(f^{-1}(x),M-f^{-1}(U))<2/m<1/n$. This is a contradiction. Thus $B(a,1/m)\subset f^{-1}(U)$, hence $f(B(a,1/m))\subset ff^{-1}(U)=U$. So $st(x,\mathcal{P}_m)\subset U$. This proves that $\{st(x,\mathcal{P}_n):n\in N\}$ is a network at x.

Claim 2. $\mathcal{P}_n$ is a cs^*-cover for every $n\in N$.

Proof. Let S be a sequence in X converging to $x\in X$. Since f is sequentially-quotient, there exists a sequence L in M converging to $a\in M$ such that $f(L)=S'$ is a subsequence of S. Then there exists $b\in M'$ such that $a\in B(b,1/n)\in\mathcal{B}_n$. Thus L is eventually in $B(b,1/n)$, so $S'=f(L)$ is eventually in $P=f(B(b,1/n))\in\mathcal{P}_n$. This proves that $\mathcal{P}_n$ is a cs^*-cover for every $n\in N$.

By the above, X has a point-star network $\{\mathcal{P}_n:n\in N\}$ consisting of countable-cs^*-covers.

(5)$\Longrightarrow$(1): Let X has a point-star network $\{\mathcal{P}_n:n\in N\}$ consisting of countable-fcs-covers.

Put $\mathcal{P}_n=\{P_\alpha:\alpha\in\Lambda_n\}$ for every $n\in N$, the topology on Λ_n is the discrete topology. Put $M=\{a=(\alpha_n)\in\Pi_{n\in N}\Lambda_n:\{P_{\alpha_n}\}$ *is a network at some* $x_a\in X\}$. Note that Λ_n is countable for every $n\in N$. Then M, which is a subspace of the product space $\Pi_{n\in N}\Lambda_n$, is a separable metric space with metric d defined as follows:

Let $a=(\alpha_n),b=(\beta_n)\in M$. If $a=b$, then $d(a,b)=0$. If $a\neq b$, then $d(a,b)=1/min\{n\in N:\alpha_n\neq\beta_n\}$.

Define $f:M\longrightarrow X$ by $f(a)=x_a$ for every $a=(\alpha_n)\in M$, where $\{P_{\alpha_n}\}$ is a network at x_a. It is not difficult to prove that f is a mapping.

Claim 1. f is a π-mapping.

Proof. Let $x\in U$ with U open in X. Since $\mathcal{P}_n$ is a point-star network of X, there exists $n\in N$ such that $st(x,\mathcal{P}_n)\subset U$. Then $d(f^{-1}(x),M-f^{-1}(U))\geqslant 1/2n>0$. In fact, let $a=(\alpha_n)\in M$ such that $d(f^{-1}(x),a)<1/2n$. Then there is $b=(\beta_n)\in f^{-1}(x)$ such that $d(a,b)<1/n$, so $\alpha_k=\beta_k$ if $k\leqslant n$. Notice that $x\in P_{\beta_n}\in\mathcal{P}_n$, $P_{\alpha_n}=P_{\beta_n}$, so $f(a)\in P_{\alpha_n}=P_{\beta_n}\subset st(x,\mathcal{P}_n)\subset U$, hence $a\in f^{-1}(U)$. Thus $d(f^{-1}(x),a)\geqslant 1/2n$ if $a\in M-f^{-1}(U)$, so $d(f^{-1}(x),M-f^{-1}(U))\geqslant 1/2n>0$. This proves that f is a π-mapping.

Claim 2. f is a pseudo-sequence-covering mapping.

Proof. Let $L=\{x_n:n\in N\}\bigcup\{x\}$ be a sequence in X converging to $x\in X$. For every $n\in N$, since $\mathcal{P}_n$ is an fcs-cover, there exists a finite subfamily $\mathcal{F}_n$ of $(\mathcal{P}_n)_x$ such that L is eventually in $\bigcup\mathcal{F}_n$. Note that $L-\bigcup\mathcal{F}_n$ is finite. There exists a finite subfamily $\mathcal{G}_n$ of $\mathcal{P}_n$ such that $L-\bigcup\mathcal{F}_n\subset\bigcup\mathcal{G}_n$. Put $\mathcal{F}_n\bigcup\mathcal{G}_n=\{P_{\alpha_n}:\alpha_n\in\Gamma_n\}$, here Γ_n is a finite subset of Λ_n. For every $\alpha_n\in\Gamma_n$, if $P_{\alpha_n}\in\mathcal{F}_n$, put $L_{\alpha_n}=L\bigcap P_{\alpha_n}$, otherwise, put $L_{\alpha_n}=(L-\bigcup\mathcal{F}_n)\bigcap P_{\alpha_n}$. It is easy to see that $L=\bigcup_{\alpha_n\in\Gamma_n}L_{\alpha_n}$ and $\{L_{\alpha_n}:\alpha_n\in\Gamma_n\}$ is a family of compact subsets of X. Put $K=\{(\alpha_n)\in\Pi_{n\in N}\Gamma_n:\bigcap_{n\in N}L_{\alpha_n}\neq\emptyset\}$. Then

(i) $K\subset M$ and $f(K)\subset L$: Let $a=(\alpha_n)\in K$, then $\bigcap_{n\in N}L_{\alpha_n}\neq\emptyset$. Pick $y\in\bigcap_{n\in N}L_{\alpha_n}$, then $y\in\bigcap_{n\in N}P_{\alpha_n}$. Note that $\{P_{\alpha_n}:n\in N\}$ is a network at y if and only if $y\in\bigcap_{n\in N}L_{\alpha_n}$. So $a\in M$ and $f(a)=y\in L$. This proves $K\subset M$ and $f(K)\subset L$.

(ii) $L\subset f(K)$: Let $y\in L$. For every $n\in N$, Pick $\alpha_n\in\Gamma_n$ such that $y\in L_{\alpha_n}$. Put $a=(\alpha_n)$, then $a\in K$ and $f(a)=y$. This proves That $L\subset f(K)$.

(iii) K is a compact subset of M: Since $K \subset M$ and $\Pi_{n\in N}\Gamma_n$ is a compact subset of $\Pi_{n\in N}\Lambda_n$. We only need to prove that K is a closed subset of $\Pi_{n\in N}\Gamma_n$. It is clear that $K \subset \Pi_{n\in N}\Gamma_n$. Let $a = (\alpha_n) \in \Pi_{n\in N}\Gamma_n - K$. Then $\bigcap_{n\in N} L_{\alpha_n} = \emptyset$. There exists $n_0 \in N$ such that $\bigcap_{n\leqslant n_0} L_{\alpha_n} = \emptyset$. Put $W = \{(\beta_n) \in \Pi_{n\in N}\Gamma_n : \beta_n = \alpha_n \ for \ n \leqslant n_0\}$. Then W is open in $\Pi_{n\in N}\Gamma_n$ and $a \in \Pi_{n\in N}\Gamma_n$. It is easy to see $W \bigcap K = \emptyset$. So K is a closed subset of $\Pi_{n\in N}\Gamma_n$.

By (i), (ii) and (iii), f is a pseudo-sequence-covering mapping and X is a pseudo-sequence-covering, π-image of a separable metric space. □

Remark 2.3. Whether "separable" and "countable-" in Theorem 2.1 can be omitted? It is still open(see [**9**, Question 3.1.14]).

3. The space with a countable sn-network

Definition 3.1. ([**3**]). Let X be a space, and let $x \in X$. A subset P of X is called a sequential neighborhood of x (called a sequence barrier at x in [**8**]) if every sequence $S = \{x_n : n \in N\} \bigcup \{x\}$ converging to x is eventually in P.

Definition 3.2. ([**11**]). Let $\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\}$ be a cover of a space X. $\mathcal{P}$ is called an sn-network of X, if $\mathcal{P}_x$ satisfies the following (a),(b) and (c) for every $x \in X$, where $\mathcal{P}_x$ is called an sn-network at x.

(a) $\mathcal{P}_x$ is a network at x.

(b) If $P_1, P_2 \in \mathcal{P}_x$, then $P \subset P_1 \cap P_2$ for some $P \in \mathcal{P}_x$.

(c) Every element of $\mathcal{P}_x$ is a sequential neighborhood of x.

Furthermore a space X with an sn-network $\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\}$ is called sn-first countable if $\mathcal{P}_x$ is countable for every $x \in X$.

Remark 3.1. In [**8**], An sn-network is called a universal cs-networks, and sn-first countable is called universally csf-countable.

Definition 3.3. ([**11**]). Let $\mathcal{P}$ be a cover of a space X.

(1) $\mathcal{P}$ is a cs-network of X, if whenever $\{x_n : n \in N\} \bigcup \{x\}$ is a sequence converging to a point $x \in U$ with U open in X, then $\{x_n : n \geqslant m\} \bigcup \{x\} \subset P \subset U$ for some $m \in N$ and some $P \in \mathcal{P}$.

(2) $\mathcal{P}$ is a cs^*-network of X, if whenever $\{x_n : n \in N\} \bigcup \{x\}$ is a sequence converging to a point $x \in U$ with U open in X, then $\{x_{n_k} : k \in N\} \bigcup \{x\} \subset P \subset U$ for some subsequence $\{x_{n_k} : k \in N\} \bigcup \{x\}$ of $\{x_n : n \in N\} \bigcup \{x\}$ and some $P \in \mathcal{P}$.

Authors of [**11**] proved that a regular space is a sequentially-quotient (or compact-covering), compact image of a separable metric space if and only if it has a countable sn-network. In ([**4**]), Ge proved that "compact" can be relaxed to "π-" here. That is, we have the following result.

Corollary 3.1. *Let X be a regular space. Then the following are equivalent.*

(1) X is a compact-covering, compact image of a separable metric space.

(2) X is a sequentially-quotient, π-image of a separable metric space.

(3) X has a countable sn-network.

The following example shows that the "regularity" in the Corollary 3.1 can not be omitted. Recall a mapping $f : X \longrightarrow Y$ is quotient if whenever $f^{-1}(U)$ is open in X, then U is open in Y.

Example 3.1. A space with a countable base is not a sequentially-quotient, π-image of a metric space.

Proof. Let R be the set of all real numbers, and let τ be the Euclidean topology on R. Put $X = R$ with the topology $\tau^* = \{\{x\} \bigcup (D \bigcap U) : x \in U \in \tau\}$, where D is the set of all irrational numbers. That is, X is the pointed irrational extension of R. Then X is Hausdorff, non-regular, and has a countable base ([**15**, Example 69]). Lin showed that X is not a symmetric space ([**9**, Example 3.13 (5)]), so X is not a quotient, π-image of a metric space ([**16**]). Note that every sequentially-quotient mapping onto a first countable space is quotient([**1**]). Thus X is not a sequentially-quotient, π-image of a metric space. □

By viewing Corollary 3.1, we have the following results without requiring the regularity of the spaces involved.

Proposition 3.1. *For a space X, the following are hold.*

(1) If X is a sequentially-quotient, π-image of a separable metric space, then X has a countable sn-network.

(2) If X has a countable closed sn-network, then X is a compact-covering, compact image of a separable metric space.

Proof. The proof of (2) is as the proof of [**11**, Theorem 4.6 (3)$\Longrightarrow$(2)], we omit it. We only to prove (1).

Let X be a sequentially-quotient, π-image of a separable metric space. Then X is sn-first countable from the proof [**4**, Theorem 2.7 (3)$\Longrightarrow$(1)], and X has a countable cs^*-network from the proof [**4**, Lemma 2.6]. We claim that X has a countable cs-network. In fact, let $\mathcal{P}$ be a countable cs^*-network. Put $\mathcal{F} = \{F = \bigcup \mathcal{P}' : \mathcal{P}' \textit{ is a finite subfamily of } \mathcal{P}\}$, then $\mathcal{F}$ is countable. It suffices to prove that $\mathcal{F}$ is a cs-network. Let $S = \{x_n : n \in N\} \bigcup \{x\}$ be a sequence in X converging to $x \in U$ with U open in X. Put $\mathcal{P}' = \{P \in (\mathcal{P})_x : P \subset U\} = \{P_n : n \in N\}$. We only need to prove that S is eventually in $\bigcup_{n \leqslant k} P_n \in \mathcal{F}$ for some $k \in N$. If for any $k \in N$, S is not eventually in $\bigcup_{n \leqslant k} P_n$, then for every $k \in N$, there exists $x_{n_k} \in S - \bigcup_{n \leqslant k} P_n$. We may assume $n_1 < n_2 < \cdots < n_{k-1} < n_k < n_{k+1} < \cdots$. Put $S' = \{x_{n_k} : k \in N\} \bigcup \{x\}$, then S' is a sequence converging to x. Since $\mathcal{P}$ is a cs^*-network, there exist $m \in N$ and a subsequence S'' of S' such that S'' is eventually in P_m. Note that $P_m \in (\mathcal{P})_x$. This contradicts the construction of S'. Thus X is an sn-first countable space with a countable cs-network $\mathcal{F}$. So X has a countable sn-network from [**8**, Theorem 3.18]. □

However, the following two questions are still open. If the answer of Question 3.1 is positive, then so is Question 3.2.

Question 3.1. *Is a sequentially-quotient, π-image of a separable metric space a sequentially-quotient, compact image of a separable metric space?*

Question 3.2. *(*[**9**, Question 3.2.12]*). Is a quotient, π-image of a separable metric space a quotient, compact image of a separable metric space?*

References

[1] J. R. Boone and F. Siwiec, *Sequentially quotient mappings*, Czech. Math. J. **26** (1976), 174–182.
[2] D. K. Burke, *Cauchy sequences in semimetric spaces*, Proc. AMS. **33** (1972), 161–164.
[3] S. P. Franklin, *Spaces in which sequence suffice*, Fund. Math. **57** (1965), 107–115.
[4] Y. Ge, *Spaces with countable sn-networks*, Comment Math. Univ. Carolinae **45** (2004), 169–176.
[5] R. W. Heath, *On open mappings and certain spaces satisfying the first countability axiom*, Fund. Math. **57** (1965), 91–96.
[6] Y. Ikeda, C. Liu and Y. Tanaka, *Quotient compact images of metric spaces, and related matters*, Topology Appl. **122** (2002), 237–252.
[7] J. A. Kofner, *On a new class of spaces and some problems of symmetrizable theory*, Soviet Math. Dokl. **10** (1969), 845–848.
[8] S. Lin, *A note on the Arens' spaces and sequential fan*, Topology Appl. **81** (1997), 185–196.
[9] S. Lin, *Point-Countable Covers and Sequence-Covering Mappings*, Chinese Science Press, Beijing, 2002. (in Chinese)
[10] S. Lin.,C. Liu and M. Dai, *Images on Locally Separable Metric Spaces*, Acta Math. Sinica, New Series **13** (1997), 1–8.
[11] S. Lin and P. Yan, *Sequence-Covering Maps of Metric Spaces*, Topology Appl. **109** (2001), 301–314.
[12] S. Lin and P. Yan, *Notes on cfp-covers*, Comment Math. Univ., Carolinae **44** (2003), 295–306.
[13] E. Michael, *$\aleph_0$-spaces*, J. Math. Mech. **15** (1966), 983–1002.
[14] V. I. Ponomarev, *Axiom of countability and continuous mappings*, Bull Pol. Acad Math. **8** (1960), 127–133.
[15] L. A. Steen and J. A. Seebach Jr, *Counterexamples in topology*, New York: Springer-Verlag, 1978.
[16] Y. Tanaka, *Symmetric spaces, g-developable spaces and g-metrizable spaces*, Math. Japon. **36** (1991), 71–84.
[17] P. Yan, *On the compact images of metric spaces*, J. Math. Study **30** (1997), 185–187. (in Chinese)

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